THE DESIGN, CONSTRUCTION AND OPERATION OF A SINGLE-PHASE CAPACITOR- START INDUCTION MOTOR

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CONTENTS

SCOPE

In the beginning of each of the chapters (except chapter Nos. 1,12,13 & 14), a short summary is given.

Starting torque is dealt at length as this occupies an important part in single phase induction motors. Harmonic theory of noise, predetermination of noise frequencies from the numbers of rotor and stator slots, and slot combination check up for less noise by the use of harmonic chart find place in the chapter on harmonics. The equivalance of the double revolving field theory and cross field theory is proved by evolving the same equivalent circuit by both the theories. The method of symmetrical components is applied to a general two phase unbalanced machine and the particular case of single phase machine is derived therefrom.

A capacitor start single phase induction motor is designed and constructed. Its performance is predetermined from the design data and compared with test results.

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B3. Fractional Horsepower Motors by Virnott.

B4. Windings of A.C. Machines by Garik.

B5. Nature of Polyphase Induction Machines by Philip Alger.

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B7. Alternating current machinery by J.G. Tarbousc.

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LIST OF SYMBOLS

N_m	=	Number of turns in the main winding.
N_s	=	Number of turns in the starting winding.
n	=	Order of stator space harmonics.
μ	=	order of rotor space harmonic.
n_{sl}	=	Order of stator tooth harmonic.
μ_{sl}	=	Order of rotor tooth harmonic.
Q_1	=	Total number of stator teeth.
Q_2	=	Total number of rotor teeth.
Z_m	=	Magnetising impedance.
x_m	=	Magnetising reactance.
r_s	=	Stator resistance.
x_s	=	rotor resistance referred to stator primary.
x_r	=	rotor reactance referred to stator primary.
r_f	=	"apparent resistance" for the forward field.
r_b	=	"apparent resistance" for the backward field.
x_f	=	"apparent reactance" for the forward field.
x_b	=	"apparent reactance" for the backward field.
k	=	The ratio of no. of turns in the starting winding to the main winding.
R_{rf}	=	Per unit forward field apparent resistance.
X_{rf}	=	Per unit forward field apparent reactance.
R_{rb}	=	Per unit backward field apparent resistance.
X_{rb}	=	Per unit backward field apparent reactance.
ϕ_M	=	Flux due to the field of the main winding.
ϕ_Q	=	Cross axis flux.

I_m = Current in the main winding.

I_s = Current in starting winding.

I_x = Current that produces cross axis flux.

I_y = Current that is effective in the main axis.

I_{m1} = Positive sequence component of stator currents.

I_{m2} = Negative sequence components of stator currents.

E_{TMy} = Transformer emf. induced in rotor bars due to main flux effective in y axis.

E_{SMx} = Speed emf. induced in rotor bars due to main flux effective in X - axis.

E_{TQx} = Transformer emf. induced in rotor bars due to the quadrature flux effective in X-axis.

E_{SQy} = Speed emf. induced in rotor bars due to quadrature flux effective in y - axis.

$$S = \frac{\text{Rotor speed}}{\text{synchronous speed}}.$$

T = Pole pitch.

C_{wm} = The winding factor for the main winding.

C_{ws} = The winding factor for the starting winding.

l_g = Air gap length.

I_r = Rotor bar current.

N_r = Number of rotor bars.

D_{er} = Diameter of end ring.

a_{er} = Area of end ring.

l_b = Length of rotor bar.

CHAPTER 1

INTRODUCTION

The electro-magnetic forces were seen to act in a number of experiments by scientists early in the Nineteenth Century and perhaps even earlier. However it was only on 17th October 1831, that Faraday discovered the electro-magnetic induction which explained these forces scientifically.

This was followed by the discovery of rotating fields by Baily and Deprex and Professor Ferraris. Then followed the great work of Nikola Tesla between 1887 and 1891 when he invented the polyphase induction motor. His researches paved the way for placing the induction motor on a sound foundation. In this connection an extract from his speech before the American Institute of Electrical Engineers may be of some interest, as this explains the theory of induction motor in short and in its original conception.

"To obtain a rotary effort in these motors was the subject of long thought. In order to secure this result it was necessary to make such a disposition that while the poles of one element of the motor are shifted by the alternate currents of the source, the poles produced upon the other elements should always be maintained in the proper relation to the former, irrespective of the speed of the motor. Such

a condition exists in a continuous current motor: but in a synchronous motor, this condition is fulfilled only when the speed is normal.

The object has been attained by placing within the ring a properly subdivided cylindrical iron core wound with several independent coils closed upon themselves. Two coils at right angles are sufficient, but a greater number may be advantageously employed. It results from this disposition that when the poles of the ring are shifted, currents are generated in the closed armature coils. These currents are the most intense at or near the points of the greatest density of the lines of force, and their effect is to produce poles upon the armature at right angles to those of the ring, at least theoretically so; and since this action is entirely independent of the speed - - that is, as far as the location of the poles is concerned -- a continuous pull is exerted upon the periphery of the armature. In many respects these motors are similar to the continuous current motors. If load is put on, the speed, and also the resistance of the motor, is diminished and more current is made to pass through the energizing coils, thus increasing the effort. Upon the load being taken off, the counter-electromotive force increases and less current passes through the primary or energizing coils. Without any load the speed is very nearly equal to that of the shifting poles of the field magnet.

"It will be found that the rotary effort in these motors fully equals that of the continuous current motors. The effort seems to be greatest when both armatute and field magnet are without any projections".

Later in 1894, the circle laws of the induction motor was proved by A. Heyland and B.A. Behrend.

Since the first announcement of Mr. Tesla that rotation can be accomplished by employing two or more alternating currents of displaced phase, the question has been raised by many an engineer as to the possibility of producing a rotary magnetic field by the use of a single circuit, carrying the ordinary alternating current, instead of requiring 3 or more wires or several independent circuits.

Prof. Elihu Thomson had claimed that he made an alternating current motor for single phase currents having a pair of single phase energizing coils surrounding the revolving iron bars (rotor), upon which latter three coils were wound. This machine, he added, with its coils closed, was without starting torque and was therefore provided with a commutator which short-circuited the coils while in proper position successively. This gave the desired starting torque. It was found that the coils after starting could be short-circuited and the commutator dispensed with. The experiments leading up to this discovery are described by him in the U.S. Patent No. 363, 185 issued to him in May 17, 1887.

The above motor, which is now known as repulsion-start induction motor, was thus reported to have been made by Prof. Elihu Thomson in 1887. Mr. Gutman had claimed to have made a similar motor in 1891, independently. Probably these were the first single phase motors.

The first split-phase self-starting motor was developed by Tesla and was used for driving small desk fans but was not employed generally for power service. Several years after this between 1893 to 1895 self-starting split-phase motors were designed with two windings in the primary one of which was employed primarily while running and the other only for the purpose of starting. A phase splitter consisting of a manually operated external switch and resistance, connected the primary windings to the supply circuit, inserting resistance in the starting winding.

Somewhat later, motors were designed with which were used starting devices supplied with a condenser in place of a resistance. About 1898 single phase induction motors were designed which started as series motors. The secondary winding was similar to that of a series motor, the commutator bars being short-circuited as the armature accelerated after which the motor ran as an induction motor.

In 1898, Dr. Steinimetz, explained different phase spliting devices which dispensed with the commutator altogether. He claimed that by then single phase induction motors in sizes upto 100 H.P. had been manufactured and sold.

Later on further developments have been made and many types of single phase motors are in use at present.

Early in this century, many questions like, good design, elimination of harmonics & noise, improved efficiency, the reduction of eddy-current losses, proper ventilation, easy speed-control, power-factor correction and many other questions which were unsolved, were tackled and better knowledge of induction machines was gained though complete success in all cannot be claimed. These were dealt with by many engineers - Alger, Kron, Chapman, Vickers, Hunt, A.B. Field, Appleman, Morril, to mention only a few; a great number of papers have been published on this and allied topics.

The induction motor is now the most widely used of all machines. It is doubtful whether the large power systems, now in such extensive use, would have been developed if this motor had not been developed. What it means to the economy of the world is appreciated by few people outside the engineering world. In the fractional horse power field, its development is phenomenal and it is applied in every form of industrial and domestic work. In this field it takes the form of single phase and three phase types.

Fractional Horse power single phase motors is very widely used in industry for oil burner, compressors, fans, blowers, office appliances and certain types of small tools; and in homes is used extensively for refrigirators, pumps,

washing and Ironing machines, room heater, vacuum cleaner, air conditioners, etc.

It has been stated that (in 1941) in U.S.A. the annual sales of fractional horse power motors of all types amounted to about $ 80,000,000 and that the rated horse power represented is of the order of 3,000,000 horse power. This indicates the importance of these motors. Not only are they important to the motor manufacturers, but also to the appliance manufacturers, the merchandisers the users and the power companies.

It is very useful in relatively small outputs. For large outputs polyphase is more advantageous as in a single phase machine (a) output is only 50% of the 3 phase motor for a given frame size and temperature rise (b) lower p.f. (c) lower efficiency and (d) has no inherent starting torque and therefore requires a starting winding with a phase-splitting device.

There are different types of single phase motors: They are:

1. Split-phase motor.
2. Resistance-start, split-phase motor.
3. Reactor-start, split-phase motor.
4. Capacitor-start motor.
5. Permanent-split capacitor motor.
6. Two-value capacitor motor.

7. Repulsion motor.
8. Repulsion-start induction motor.
9. Repulsion-induction motor.
10. Universal motor.
11. Shaded-pole motor.
12. Split-phase reluctance motor.
13. Capacitor-type reluctance motor.
14. Capacitor-type hysteresis motor.
15. Shaded pole hysteresis motor.
16. Induction Motor.

1. <u>Split-phase Motor</u>:

The following are the difinition according to ASA:

A split-phase motor is a single-phase induction motor equipped with an auxiliary winding, displaced in magnetic position from, and connected in parallel with, the main winding. (Note: Unless otherwise specified, the auxiliary circuit is assumed to be opened when the motor has attained a predetermined speed. The term "split-phase motor", used without qualification, describes a motor to be used without impedance other than that offered by the motor windings themselves, other types being separately defined).

2. <u>Resistance-start, Split-phase motor</u>:

A resistance-start motor is a form of split-phase motor having a resistance connected in series with the auxiliary winding. The auxiliary circuit is opened when the

motor has attained a predetermined speed.

3. <u>Reactor-start, Split-phase motor</u>:

A reactor-start motor is a form of split-phase motor designed for starting with a reactor in series with the main winding. The reactor is short-circuited, or otherwise made ineffective, and the auxiliary circuit is opened when the motor has attained a predetermined speed.

A <u>capacitor motor</u> is a single-phase induction motor with a main winding arranged for direct connection to a source of power and an auxiliary winding connected in series with a capacitor. (Note: The capacitor may be connected into the circuit through a transformer and its value may be varied between starting and running).

4. <u>Capacitor-start motor</u>:

A capacitor-start motor is a capacitor motor in which the capacitor phase is in the circuit only during the starting period.

5. <u>Permanent-split capacitor motor</u>:

A permanent-split capacitor motor is a capacitor motor having the same value of capacitance for both starting and running conditions.

6. <u>Two-valve capacitor motor</u>:

A 2-valve capacitor motor is a capacitor motor using

different values of effective capacitance for the starting and running conditions.

7. <u>Repulsion motor</u>:

A repulsion motor is a single-phase motor which has a stator winding arranged for connection to a source of power and a rotor winding connected to a commutator. Brushes on the commutator are short-circuited and are so placed that the magnetic axis of the rotor winding is inclined to the magnetic axis of the stator winding. This type of motor has a varying-speed characteristic.

8. <u>Repulsion-start induction motor</u>:

A repulsion-start induction motor is a single-phase motor having the same windings as a repulsion motor, but at a predetermined speed the rotor winding is short-circuited or otherwise connected to give the equivalent of a squirrel-cage winding. This type of motor starts as a repulsion motor but operates as an induction motor with constant-speed characteristics.

9. <u>Repulsion-induction motor</u>:

A repulsion-induction motor is a form of repulsion motor which has a squirrel-cage winding in the rotor in addition to the repulsion motor winding. A motor of this type may have either a constant-speed or varying-speed

characteristic.

10. Universal motor:

A universal motor is a series-wound or a compensated series-wound motor which may be operated either on direct current or single-phase alternating current at approximately the same speed and output. These conditions must be met when the direct-current and alternating-current voltages are approximately the same and the frequency of the alternating current is not greater than 60 cycles per second.

11. Shaded-pole motor:

A Shaded-pole motor is a single-phase induction motor provided with an auxiliary short-circuited winding or windings displaced in magnetic position from the main winding.

12. Split-phase reluctance motor:

The ASA does not define this type of motor, but does define the reluctance type motor.

A reluctance type motor is a synchronous motor similar in construction to an induction motor in which the member carrying the secondary circuit has salient poles without direct- current excitation. It starts as an induction motor but operates normally at synchronous speed (ASA 10.10.305).

From this definition, a split-phase reluctance motor

may be described as a reluctance-type motor, the stator of which is exactly the same as in a split-phase induction motor, i.e., the stator has a main winding, an auxiliary winding and also a starting switch.

This motor is also called unexcited synchronous motor. The rotor has a salient-pole construction. Starting torque is produced by currents in the rotor bars, i.e. cage winding provided in pole faces. As the speed nears synchronism, a value is attained at which there is sufficient synchronising action to pull the rotor quickly into step with the rotating field. Such motors vibrate more than a split-phase motor because of the salient pole rotor.

13. Capacitor-type reluctance motor:

From the ASA definition 10.10.305 given above, the capacitor type reluctance motor is a reluctance type motor the stator of which may be wound in one of three ways discussed under definitions 4,5, and 6, a-bove, that is, as capacitor-start, as permanent-split capacitor, or as 2-value capacitor.

14. Capacitor-type hysteresis motor:

A hysteresis motor is a synchronous motor without salient poles and without direct-current excitation, which starts by virtue of the hysteresis losses induced in its hardened steel secondary member by the revolving field of

the primary and operates normally at synchronous speed due to the retentivity of the secondary core (ASA 10.10.315).

From this definition, a capacitor-type hysteresis motor may be described as a hysteresis-type motor with a stator winding which may be wound in one of the three ways described under 4, 5 and 6 above.

This motor has a smooth cylindrical rotor of steel such as would be used for a permanent magnet as defined above. The rotating mmf. produced by the stator winding magnetises the rotor along an axis perpendicular to the rotor surface as the mmf. sweeps on, the retentivity of the rotor steel causes its magnetic axis to lag behind that of the mmf. As a result torque is produced that causes the rotor to follow in the direction of the mmf. If the rotor requires no torque it would come up to synch. speed and run with its magnetic axis aligned with that of the mmf. Addition of load to the motor causes the rotor to drop back in phase position in the manner that an excited synch. motor adjusts itself to load.

15. Shaded-pole hysteresis motor:

From ASA definition 10.10.315 given above, a shaded-pole hysteresis motor may be defined as a hysteresis-type motor with stator windings the same as in the shaded-pole motor defined under 11 above.

16. Induction motor:

For most types of single phase motors some special means of providing starting torque is required. The single-phase synchronous inductor motor not only has starting torque but if it is stalled when running in one direction it will at once reverse and run in the opposite direction.

The rotor laminations are stacked on a cylindrical Alnico magnet that had been magnetised radially from the centre. The periphery of the rotor has one magnetic polarity and the centre has the opposite magnetic circuit. The stator laminations have salient poles. Each of these have been notched with 5 teeth. These are so located that when the rotor and stator teeth line up under one pole, the teeth are ½ of a tooth - pitch out of line under each adjacent pole. On each of the pole cores is a coil. These are connected in series, the directions of the winding being opposite an adjacent core as shown. Assume that flux is entering all the rotor teeth, then they are all south-poles, when there is no voltage applied to stator winding. Each rotor tooth under the r.h.s pole has equal and oppositely directed horizontal components of pull acting on it. Let an alternating current be established; at the instant when the current is in the direction shown in Fig. the mmf. of the l.h.s. pole opposes the mmf. of the Alnico-magnet and the flux in that part is reduced and the mmf. of the winding on the r.h. pole aids the mmf. of the alnico magnet and the

flux passing between teeth under that pole is increased.

Now the balance of horizontal components of pull on the teeth is so unstable that in most cases the rotor will start in one direction or the other. As soon as it moves in one direction, the horizontal component of pull in that direction rapidly exceeds the component in the other and the rotor advances one-half tooth pitch in one half cycle.

REFERENCES:

B1, B2, B3, B4, B5, J1, J2, J3, J4 and J5.

CHAPTER 2

STARTING TORQUE

A poly-phase induction motor has a starting torque. A single phase induction motor has no starting torque. This is one of the disadvantages of the single phase motors and hence a great deal of attention has been paid to the methods of starting as much as to the actual operating characteristics. In fact Dr. Steinmetz's original paper on Single Phase induction motor deals more about starting than about the operation of the motor. Hence this subject of starting is discussed in detail.

First of all, it is shown fundamentally that a single phase induction motor has no inherent starting torque. The starting torque of a polyphase induction machine is due to its rotating field which is present even when the motor is at rest. Hence the subject of rotating field and how to produce such a field from a single phase supply is then taken up. This is followed by a brief description of the methods of starting. The design of the starting impedances and starting winding turns is then taken up.

In the last pages of this chapter, the specific topic of the capacitor start motor is dealt with. Its advantages over the split-phase motor are narrated, the purposes of the starting switch are then explained. Mr. Veinott's analysis (on the basis of the build-up of voltage in an induction generator) to show what happens if the starting switch

of a capacitor-start motor were to reclose presents an interesting study.

I. <u>To show that there is no inherent starting torque in a Single-phase machine.</u>

It has been explained in the chapter on the "Theories of Operation" that the single phase motor has no starting torque of its own - By the 2 revolving field theory: at starting the two oppositely revolving fields are equal in magnitude and so the torques are equal and opposite to give a net zero starting torque. By cross field theory: when the rotor is at rest, there is no cross flux at all and hence there is no torque. (The rotor behaves like the short-circuited secondary of a transformer). Further this can be shown from fundamentals as below:

It is assumed that the stator winding is of the concentric distributed type and also that the rotor is of squirrel cage construction. A result of the first assumption is that a sinusoidal flux distribution exists in the air gap. A pair of rotor conductors, situated one pole pitch apart is shown in figure (1). At any point along the gap (Fig. 1) the value of the instantaneous flux density is

$$B = B_m \sin wt \sin \frac{x}{T}\pi$$

where B_m = max. flux density.

T = pole pitch ; w = $2\pi \times$ frequency of the supply.

When the flux through an elementary strip of width dx is (when L is the axial length of coil side)

$$d\phi = B_m L \sin wt \sin \frac{x}{T}\pi \, dx$$

Then the total flux through the coil shown is:

$$\phi = B_m L \sin wt \int_{x_1}^{x_1+T} \sin \frac{x}{T}\pi \, dx$$

$$= \frac{2TL}{\pi} B_m \sin wt \cos \frac{x_1}{T}\pi$$

If the rotor is at rest, the emf. induced in the coil is,

$$e = \frac{d\phi}{dt} 10^{-8}$$

$$= \frac{2w\,TL}{\pi} B_m \cos wt \cos\left[\frac{x_1}{T}\pi\right] 10^{-8} \text{ volts.}$$

$$= V_m \cos \pi \frac{x_1}{T} . \cos wt.$$

where $V_m = \frac{2w\,TL}{\pi} B_m . 10^{-8}$ volts.

V_m is the maximum value of the emf. which can be induced in the coil.

If r and l represent the resistance and inductance of the coil, the current therein is (when the coil is shorted on itself as in a squirrel cage winding.).

$$i = \frac{V_m}{\sqrt{r^2 + w^2 l^2}} \cos \frac{x_1}{T}\pi \cos (wt - \phi)$$

where $\phi = \tan^{-1} \frac{wl}{r}$

The tangential pull on the coil is

$$P = 2 \frac{BiL}{10} \text{ dynes}$$

$$= \frac{2L}{10} \frac{V_m}{\sqrt{r^2+w^2l^2}} \cos \pi\frac{x_1}{T} \cos (wt - \phi) B_m \sin wt \sin \frac{x_1}{T}\pi$$

$$= \frac{2L}{10} \frac{V_m}{\sqrt{r^2+w^2l^2}} \tfrac{1}{2} \sin \frac{2x_1}{T}\pi \sin wt. \cos (wt - \phi)$$

$$= \frac{P_m}{2} \sin \frac{2x_1}{T}\pi \sin wt. \cos (wt - \phi)$$

(where $P_m = \frac{2L}{10} \frac{V_m B_m}{\sqrt{r^2+ w^2l^2}}$)

$$= \frac{P_m}{2} \sin \pi\frac{2x_1}{T} . \tfrac{1}{2} \left[\sin (2wt - \phi) + \sin \phi\right]$$

The mean value of P over a cycle is

$$P_{av} = \frac{P_m}{4} \sin \frac{2x_1}{T}\pi \sin \phi \text{ dynes.}$$

From this it can be seen that the average tangential pull on a coil and the torque which it contributes to the rotor is a function of its position x.

Let there be a large No. of coils with n coils per unit length. Then the pull on coils within a width dx is,

$$\frac{P_m}{4} n \sin\theta \sin \frac{2x}{T}\pi \, dx.$$

If the rotor is assumed to be at rest then x is not a function of t. Then the average pull due to conductors over one pole pitch on the rotor is:

$$\frac{P_m}{4} n \sin\theta \int_0^T \sin \frac{2x}{T}\pi \, dx$$

$$= \frac{P_m}{4} n \frac{T}{2\pi} \sin\theta \left[- \cos \frac{2x}{T} \right]_0^T$$

$$= 0$$

Hence, in a single phase motor of which the rotor is at rest, the resultant torque is zero.

II. Rotating Field.

The starting torque in a polyphase machine is due to its revolving fieldwhich is present even when the rotor is at standstill. There is no revolving field when a single phase motor's rotor is at rest, hence there is no starting torque. Therefore, the starting problem can be solved if we can find any method to produce a revolving field from a single phase supply.

It has been shown by Deprex that a 2 phase supply can be used to produce a rotating field. In 1883 he fed alternating current to a coil which produced an alternating or

oscillating field along the OX axis. He supplied another coil whose magnetic axis made an angle of 90° with the OX axis with a.c. whose phase difference was 90° in time from the current in the first coil and showed that a revolving field of constant amplitude could be produced. (The frequency of the two currents are the same and the number of turns in each coil was the same).

He also showed that if the two currents were of equal period, but not of equal amplitude, an elliptically rotating field was produced.

Further it can be shown analytically that a crude form of rotating field can be produced by having two coils at a space displacement of 90 degrees and sending currents which are having ω degrees time phase displacement.

(a) <u>Rotating field produced due to space displacement $\pi/2$ and any time displacement:</u>

Referring to figure let ϕ_1 & ϕ_2 represent two single phase alternating fields displaced in time and space phases by ω and by φ respectively. Then instantaneously we have,

$$\phi_1 = \phi_1 \sin wt. \quad \phi_2 = \phi_2 \sin (wt - \omega).$$

The perpendicular horizontal comp. of ϕ_2, are respy.

$$\phi_2 \sin (wt - \omega) \cos \varphi \text{ and } \phi_2 \sin (wt - \omega) \sin \varphi$$

Hence the total horizontal field $= \phi_2 \sin (wt - \omega) \sin \varphi$

Hence the total vertical field $= \phi_1 \sin wt + \phi_2 \sin(wt - \alpha)\cos\psi$

$\therefore$ The resultant field is, ϕ_r,

$$\phi_r = \sqrt{\phi_2^{\,2}\sin^2(wt-\alpha)\sin^2\psi + \phi_1^2 \sin^2 wt + \phi_2^2 \sin^2(wt-\alpha)\cos^2\psi + 2\,\phi_1\phi_2 \sin wt\, \sin(wt-\alpha)\cos\psi}$$

The angle which ϕ_r makes with the horizontal is given by

$$\tan^{-1} \frac{\phi_1 \sin wt + \phi_2 \sin(wt-\alpha)\cos\psi}{\phi_2 \sin(wt-\alpha)\sin\psi}$$

When $\psi = 90^\circ$, i.e. when the space angle between two fields is 90°,

$$\phi_r = \sqrt{\phi_2^{\,2} \sin^2(wt-\alpha) + \phi^2{}_1 \sin^2 wt}$$

$$\lambda = \tan^{-1} \frac{\phi_1 \sin wt}{\phi_2 \sin(wt-\alpha)}$$

$$\therefore \tan\lambda = \frac{\phi_1 \sin wt}{\phi_2 \sin(wt-\alpha)}$$

The angular velocity with which this resultant flux rotates

$$= \frac{d\lambda}{dt}$$

Now, $\sec^2 \dfrac{d\lambda}{dt} =$

$$\frac{\phi_2 \sin(wt-\alpha)\,\phi_1 w \cos wt - \phi_1 \sin wt\, \phi_2 w \cos(wt-\alpha)}{\phi_2^{\,2} \sin^2(wt-\alpha)}$$

$$= \frac{w\, \phi_1 \phi_2 \quad \sin(wt - \alpha) \cos wt - \sin wt \cos (wt - \alpha)}{\phi_2^2 \sin^2 (wt - \alpha)}$$

$$\tan \lambda = \frac{\phi_1 \sin wt}{\phi_2 \sin^2 (wt - \alpha)} \quad ;$$

$$1 + (\tan \lambda)^2 = \frac{\phi_2^2 \sin^2 (wt - \alpha) + \phi_1^2 \sin^2 wt}{\phi_2^2 \sin^2 (wt - \alpha)}$$

$$= \sec^2 \lambda$$

$$\text{Therefore,} \quad \frac{d\lambda}{dt} = \frac{w\, \phi_1 \phi_2 (\sin \overline{wt - \alpha} \cos wt - \sin wt \cos \overline{wt - \alpha})}{\phi_2^2 \sin^2 (wt - \alpha)}$$

$$\frac{\phi_2^2 \sin^2 (wt - \alpha)}{\phi_2^2 \sin^2 (wt - \alpha) \quad \phi_1^2 \sin^2 wt}$$

$$= \frac{w\, \phi_1 \phi_2}{\phi_r^2} (\sin \overline{wt - \alpha} \cos wt - \sin wt \cos \overline{wt - \alpha})$$

From the above expressions for ϕ_r and $\frac{d\lambda}{dt}$, it can be seen that these are periodic functions of time - i.e. ϕ_r the resultant field rotates, varying not only its amplitude but also its speed. A crude form of rotating field is produced. For 2 phase conditions, $\alpha = \pi/2$ and $\phi_2 = \phi_1$. Then $\phi_r = \phi_1$ and $\frac{d\lambda}{dt} = -w$; i.e., a true rotating field is produced as already stated.

(b) Methods of starting:

Hence it is clear that in order to have a rotating field we must have another winding and send through it a current which has a time phase displacement with the current

in the main winding. This winding may be called STARTING WINDING or AUXILIARY WINDING. This winding can be advantageously placed at 90° electrical space angle from the main winding or running winding. In order to obtain current having a time phase displacement with the current in the main winding, a method was suggested by Prof. Ferraris. In 1888 in his paper on "Electrodynamic Rotations produce by means of Alternating Currents" he suggested the methods of obtaining currents, differing in phase nearly 90 deg., by inserting a resistance in one winding and inductance in the other, thus making the ratio of reactance/resistance small in one winding and large in the other. This is the basis of starting methods of single phase induction motors now adopted widely. (However the first split-phase self-starting single-phase motor was developed by Tesla as already stated in the chapter 1).

Three different methods now adopted on this basis are indicated below:

(1) Connecting capacitor in series with the starting winding called CAPACITY SPLIT PHASE.

(2) Connecting resistance in series with starting winding called RESISTANCE SPLIT PHASE and

(3) Connecting inductance in series with the main winding called INDUCTANCE SPLIT PHASE.

The schematic diagrams for the three different methods

of starting are shown along with the vector diagram, for the voltage and currents in the windings. The space axes of the two windings are 90 degrees apart in all cases. The time phase angle between the two currents is shown as α. In the case of the capacity type of motor, this angle may be made 90 electrical degrees. In the resistance and inductance types the starting winding must have a high resistance, while in the capacitor type such a high resistance winding is not necessary. A centrifugal switch is shown in the starting winding. This is opened after the motor has attained normal speed. In the inductance motor it is desirable to short circuit the starting inductance which appears in the running winding.

Apart from the split phase starting described above, two other typical starting methods are:

(4) by the use of a small starting motor.

This is suitable with large single phase motor which does not have to start under load. A small self starting motor is directly coupled to the large main motor. Starting is obtained with the small starting motor and when the set comes upto normal speed, the power supply is connected to the main motor and disconnected from the starting motor.

(5) By the application of the repulsion motor as the starting device:

This is called the repulsion start induction run motor. In fact this method has been adopted in the single phase motors built up for the first time by Prof. Elihu Thomson. This motor has been defined in the first chapter. Here in this thesis this method will be passed up without much discussion. This section on the methods of starting will not be complete without mentioning the recent develop -ments made in this field - the development of single-winding single-phase self-starting motors. This has been made possible by the use of assymmetry to obtain starting torque in a single phase motor with a single winding.

(6) The Oswald motor developed by Mr. Earl Oswald has more turns about one side of the pole than the other, producing unequal saturation. Since this saturation is in part the result of both stator and rotor leakage flux, accompanying the starting current under lower full-load currents the saturation is reduced and normal running conditions are made possible.

(7) The asymmetry can also be achieved by using varying air gap. The varying-air gap motor had been manufactured by Bodine Electric Company of Chicago. It has been stated that the difference in permeability resulting from higher flux densities where the air-gap is lower will not be serious.

Such self-starting single-winding single-phase motors are economically and technically desirable because of the

simplicity in avoiding a separate starting winding and switch and, therefore, giving an attendant saving in manufacturing cost. Another advantage, is the avoidance of maintenance troubles and outages, since by far the greatest single factor causing breakdowns in single phase motors is trouble with starting winding and its automatic switch.

III. Starting torque.

The actual evaluation of the starting torque is not attempted here; this can be found in the chapters on the theories of operation. However the important factors on which the starting torque depends are analysed hereunder with a view to enables us to determine starting impedances and starting windings of the single-phase induction motors.

It has been shown earlier that the resultant revolving field due to two windings at 90^o electrical space angle with currents displaced by degrees time angle is of a magnitude ϕ_r which is a function of time t;

$$\phi_r = \sqrt{\phi_2{}^2 \sin^2 (wt - \alpha) + \phi_1{}^2 \sin^2 wt}$$

and that its angular velocity which is also a function of time t is

$$\frac{d\lambda}{dt} = \frac{w\,\phi_1\phi_2}{\phi_r{}^2}(\sin \overline{wt - \alpha} \cos wt - \sin wt \cos \overline{wt - \alpha})$$

Let M_r be the mmf. corresponding to the flux ϕ_r.
If the flux is proportional to the mmf. then the current

induced in the rotor is proportional to $M_r \dfrac{d\lambda}{dt}$ (Here the change in mag. of M_r with time is being neglected.)

Hence the torque is proportional to $M_r^2 \dfrac{d\lambda}{dt}$

$$\frac{d\lambda}{dt} = wM_1M_2 \frac{(\sin \overline{wt - \theta} \cos wt - \sin wt \cos \overline{wt - \theta})}{M_r^2}$$

Therefore,

$$T \propto M_r^2 \frac{d\lambda}{dt} \quad wM_1M_2(\sin \overline{wt - \theta} \cos wt - \sin wt \cos \overline{wt - \theta})$$

$$\propto w M_1M_2 \left\{ \tfrac{1}{2} \left[\sin \overline{2wt - \theta} + \sin(-\theta) \right] - \tfrac{1}{2} \left[\sin \overline{2wt - \theta} \sin\theta \right] \right\}$$

$$\propto w M_1M_2 \sin\theta .$$

Under starting conditions, an induction motor behaves as a short circuited transformer possessing a large air gap. With the assumption of a resistance-less rotor, practically all the flux is leakage flux and is proportional to and in phase with the stator current. Hence the mmf.'s in the equations above are proportional to their turns and currents while the cts are inversely proportional to their respective winding impedances. The impedances under these conditions are termed equivallent i.e. they are given by the quotients of the volts by currents and repr. impedance of the motor as a whole referred to the stator windings.

Hence the starting torque $T_s \propto N_sN_m \underline{I}_s\underline{I}_m \sin\theta \propto \dfrac{N_sN_m}{Z_sZ_m} \sin\theta$

Therefore $T_s = K_I\underline{I}_s\underline{I}_m \sin\theta$.

or $T_s = K_c \dfrac{N_s N_m}{Z_s Z_m} \sin$ where K_I and K_c are constants of proportionality.

Starting impedances for Maximum starting torque:

In the three cases considered below (resistance, capacitor & inductance start), it is assumed that both the windings have been designed and only the starting impedance to be connected in series with one of the windings is to be determined to give maximum torque.

(a) Resistance start: The complete circuit of this motor is shown in Fig. 5. The current drawn by the running winding is $\underline{I}_m$ and it lags voltage V_s by the p.f. angle ϕ_m. Since $\underline{Z}_m$ is fixed, the current $\underline{I}_m$ is also constant. The starting winding has an external resistance R which can be varied.

The envelope of a series circuit with constant reactance and variable resistance is a circle having a diameter equal to the voltage divided by the reactance. The total current is the vector sum of $\underline{I}_r$ and $\underline{I}_s$ i.e. the distance OC, in the figure. (Note that the size of the circle is independent of the resistance R).

It has just been shown that the starting torque is $\propto \underline{I}_s \; I_m \sin\alpha$. However $\underline{I}_m$ is fixed.

$\therefore T_s \propto I_s \sin\alpha \propto$ CD.

Hence the maximum torque is obtained when CD is a maximum, which can be got by drawing a parallel to OA tangential to the circle; the maximum length of CD is thus MN. Since, M is the mid-point of arc AMG,

$$\theta_s = \frac{\theta_m}{2}$$

$$\therefore \quad \cot\theta_s = \cot\frac{\theta_m}{2} = \frac{(\cos\theta_m)/2}{(\sin\theta_m)/2}$$

$$= \frac{\cos^2\frac{\theta_m}{2}}{\sin\frac{\theta_m}{2}\cos\frac{\theta_m}{2}} = \frac{1+\cos\theta_m}{\sin\theta_m}$$

$$\therefore \quad \cot\theta_s = \frac{1+\cos\theta_m}{\sin\theta_m} \qquad \text{L.H.S.} = \frac{R+r_s}{x_s}$$

$$\text{R.H.S} = \frac{1+\frac{r_m}{z_m}}{\frac{x_m}{z_m}}$$

$$= \frac{z_m+r_m}{x_m}$$

Therefore $\dfrac{R+r_s}{x_s} = \dfrac{z_m+r_m}{x_m}$

Therefore the external resistance R, for maximum torque is:

$$R = \frac{x_s}{x_m}\left[z_m+r_m\right] - r_s$$

(b) Capacity Starting:

The circuit diagram for the capacity start motor and

the corresponding starting circle diagram are shown in figure 6. The starting current I_s will have a circle envelope, the circle diameter being $= \frac{V_s}{r_s}$ the diameter being parallel to V_s. Starting torque is proportional to $I_s \sin\phi$, I_m being fixed. $I_s \sin = MN$. To locate the pt M, draw a line tangent to the circle and parallel to the current I_m.

$\angle HAM = \angle MAG$ i.e. $90 - \phi_s = \phi_s + \phi_m$

$$\therefore \quad s = \frac{90 - \phi_m}{2}$$

$$\therefore \quad \tan \phi_s = \tan \frac{90 - \phi_m}{2} = \frac{\sin(90 - \phi_m)}{1 \quad \cos(90 - \phi_m)}$$

$$= \frac{\cos \phi_m}{1 + \sin \phi_m}$$

Therefore $\tan \phi_s = \frac{\cos \phi_m}{1 + \sin \phi_m}$ L.h.s $= (x_c - x_s)/r_s$

$$\text{R.h.s} = \frac{r_m/z_m}{1 + \frac{x_m}{z_m}} = \frac{r_m}{z_m + x_m}$$

Therefore, $\frac{x_c - x_s}{r_s} = \frac{r_m}{z_m + x_m}$

$\therefore$ For maximum starting, the starting winding must have an external condenser with a capacity equal to

$$x_c = x_s + \frac{r_m r_s}{z_m + x_m}$$

(c) Inductance start:

The circuit for a split phase motor with external inductance in the running winding and the corresponding circle diagram are shown in the figure. 7. Again, the point for maximum starting torque is determined by drawing a line tangent to the circle and parallel to the current vector $\underline{I}_s$ as shown at M.

Now $\angle KAM = \angle GAM$, i.e. $\theta_m - \theta_s = 90 - \theta_m$

$$\therefore \quad \theta_m = \frac{90 + \theta_s}{2}$$

$$\therefore \quad \tan \theta_m = \tan\left(\frac{90 + \theta_s}{2}\right) = \frac{\sin(90 + \theta_s)}{1 + \cos(90 + \theta_s)}$$

$$\text{Therefore} \quad \tan \theta_m = \frac{\cos \theta_s}{1 - \sin \theta_s}$$

$$\therefore \quad \frac{x_\ell + x_m}{r_m} = \frac{r_s}{z_s - x_s}$$

$\therefore$ For maximum starting torque the running winding should have an external inductive reactance:

$$x_\ell = \frac{r_m r_s}{z_s - x_s} - x_m$$

STARTING WINDING FOR MAXIMUM STARTING TORQUE:

In the preceding section starting impedances to be connected to one of the windings (both of which have already been fixed) have been determined. However, the more practical cases are the design of the starting winding

itself of a split-phase motor and the starting winding and the capacitor of a capacitor-start motor. These will now be considered.

The determination of the ratio of turns, K, is an important one in the case of split-phase motor. (This motor has been defined in Chapter 1 to be one which is used without impedance other than that offered by the motor windings themselves). This ratio is determined in this section to give a maximum starting torque, with the line current not exceeding a predetermined value.

In the case of the capacitor-start motor both the starting winding and the capacitor are to be determined. Here the ratio of slot space alloted to the two windings is assumed to be fixed and the value of the capacitor is determined for optimum starting torque. It is shown that for the optimum starting torque, the ratio of turns should be small; but a small ratio of turns results in the necessity of having a large capacitance. Hence a compromise has to be found out and then the value of K fixed. This aspect is also dealt with in what follows:

(a) Turns ratios for maximum starting torque in the split-phase motor.

In order to produce a ph. displacement between currents, the reactance x_s is made less than x_r. This is effected by having less no. of turns in the starting

winding than in the running winding. In order to limit the current in z_s and to increase further , the resistance r_s is made relatively high by winding the starting winding with wire of smaller across section than that of running winding. This also has the result of economizing the winding space.

$$\sin \quad = \sin(\phi_m - \phi_s) = \sin\phi_m \cos\phi_s - \cos\phi_m \sin\phi_s$$

$$= \frac{x_m}{z_m} \cdot \frac{r_s}{z_s} - \frac{r_m}{z_m} \cdot \frac{x_s}{z_s} = \frac{(x_m r_s - r_m x_s)}{z_m z_s}$$

But $T_s = \dfrac{k_c N_m N_s}{z_s z_m} \sin\phi$

$= \dfrac{k_c N_m N_s}{z_m^2 z_s^2} (x_m r_s - r_s x_s)$ all equivalent values.

In order to find the optimum value of this expression, some limiting condition or conditions must be assumed. Because the starting ct of a split phase motor is relatively high, it is desirable to take the condition that the starting ct is not to exceed a given value*

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* In fact NEMA standards on fractional h.p. motors prescribes ceiling values for starting current (the figures being taken with the rotor locked). Though these are given under the chapter on specifications they are quoted here also:-

Starting current:- Maximum values of starting current are laid down. The values are given below for 230 volts motors irrespective of frequency.

H.P.	1/6 or less	1/4	1/3	1/2	3/4
Locked amps.	10	11.5	15.5	22.5	30.5

Let the ratio between the line starting ct and that in the running winding be $= k_1$ (note that k_1 is now a given condition hence fixed once for all - it is not variable). Then since the ph. angle between the starting currents in the two windings is usually such that these currents may be added arithematically;

$$\frac{(V/z_s\ z_m)(Z_s + z_m)}{V/z_m} = k_1 = \frac{(z_s + z_m)\ z_m}{z_s\ z_m}$$

$$= 1 + \frac{z_m}{z_s}$$

Therefore $\dfrac{z_m}{z_s} = k - 1$

$\therefore\ z_s = z_m \left(\dfrac{1}{k-1} \right) = z_m k_2$ (k_2 is fixed since k is fixed).

If we assume now the turns ratio $\dfrac{N_s}{N_m} = k$

Then $x_s = k^2\ x_m$; $N_s = k\ N_m$

$\therefore$ The equation for the starting torque becomes;

$$T_s = \frac{k_c\ N_s\ N_m}{z_s^2\ z_m^2} (x_m\ r_s - r_m\ x_s)$$

$$= \frac{k_c\ k\ N_m^2}{(k_2^2\ z_m)^2\quad z_m^2} \left[(x_m\ r_s) - (k^2\ x_m\ r_m) \right]$$

$$= \frac{k_c\ k\ N_m^2}{k_2^4\ z_m^4} (x_m r_s - k^2\ x_m\ r_m)$$

k is the variable which has to be fixed for max. torque from the above equation. (The running winding being completely designed and fixed all quantities with suffix m are fixed; further $z_s = k_2 z_m$; hence z_s is also fixed).

Hence, $T_s \propto k\,(x_m r_s - k^2 x_r r_m)$

$$\propto x_r r_s k - k^3 x_m r_m$$

$$r_s = (z_s^2 - x_s^2)^{\frac{1}{2}} = (z_s^2 - k^4 x_m^2)^{\frac{1}{2}}$$

$$\therefore T_s \propto k \; x_m \; (z_s^2 - k^4 x_m^2)^{\frac{1}{2}} - k^3 x_m r_m$$

Differentiating with reference to k ,

$$\frac{dT_s}{dk} = k\, x_m \tfrac{1}{2} (z_s^2 - k^4 x_m^2)^{-\frac{1}{2}} (-x_m^2\, 4k^3) \quad x_m (z_s^2 - k^4 x_m^2)^{\frac{1}{2}}$$
$$- 3k^2 x_m r_m = 0$$

This reduces to $r_s^2 - 2\, x_s^2 - 3k^2\, r_m\, r_s = 0$

Since the p.f. in the starting winding is near unity

$r_s \simeq z_s$ & x_s^2 is small compared to r_s^2.

$$\therefore z_s^2 = 3\, r_m z_s\, k^2 \qquad \therefore k = \sqrt{\frac{z_s}{3r_m}} = \sqrt{\frac{k_2 z_m}{3\, r_m}}$$

$$\therefore N_s = N_m \sqrt{\frac{k_2 z_m}{3 r_m}}$$

(b) <u>Capacity start motor: determination of starting winding & capacitance.</u>

A much greater starting torque for the same line starting current may be obtained if a capacitor is placed in series with the starting winding. The net reactance of the starting winding is $(x_s - x_c)$; starting torque is then given by:

$$T_s = \frac{k_c \; N_s \; N_m}{z_m^2} \left[\frac{x_m \, r_s - r_m(x_s - x_c)}{r_s^2 \quad (x_s - x_c)^2} \right]$$

where x_c is the reactance of the capacitor. If A_m and A_s are, respectively, the cross-sectional areas of the total spaces available for the two windings, and if these areas are fully occupied, then

$$r_m = \frac{N_r \, \ell_m \sigma}{A_m / N_r} + \left[\frac{N_m}{N_r} \right]^2 r_r$$

$$r_s = \frac{N_s \, \ell_s \sigma}{A_s / N_s} + \left[\frac{N_s}{N_r} \right]^2 r_r$$

where and are respectively the mean length of turn and specific resistance of the winding material, N_r the number of turns of the rotor winding and r_r the rotor resistance. If it is assumed that the two terms in each right hand member are equal, then

$$\frac{r_s}{r_m} = \frac{A_m \; N_s^2}{A_s \; N_m^2} = k_3 \; (k)^2$$

where k_3 is the ratio of the No. of slots of the running winding to those of the starting winding (This assumes that the windings occupy different slots and that $\ell_m = \ell_s$).

k is the ratio of turns. Substituting for r_s and x_s in the starting torque equation,

$$T_s = \frac{k_c N_m^2 r_m}{z_m^2}\left[\frac{k_3 k^3 x_m - k\ (k^2 x_m - x_c)}{k_3^2 k^4 r_m^2 + (k^2 x_m - x_c)^2}\right]$$

$$= \frac{k_c N_m^2 r_m}{z_m^2}\left[\frac{(k_3 - 1)k^3 x_m + k x_c}{k_3^2 k^4 r_m^2 + (k^2 x_m - x_c)^2}\right]$$

Writing $a = (k_3 - 1)k^3 x_m$

$b = k$. $c = k_3^2 k^4 r_m^2$. $d = k^2 x_m$

$$T_s = \frac{a + bx_c}{c + (d - x_c)^2}$$

$$\therefore \frac{dT_s}{dx_c} \propto \frac{b\left[c + (d - x_c)^2\right] + 2(a + bx_c)(d - x_c)}{\left[c + (d - x_c)^2\right]^2} = 0$$

for a maximum. Substituting and simplifying, we have, finally,

$$x_c = k^2\left[k_3 z_m + (1 - k_3)\ x_m\right]$$

Substituting this value of x_c in the starting torque equation,

$$T_s = \frac{k_c N_m^2 r_m}{2k\ k_3\ z_m^2}\left[\frac{1}{z_m - x_m}\right]$$

k_3 is the ratio of slots alloted to windings and is fixed. Hence x_c is proportional to k. Since x_c $1/c$ in order to keep c small, x_c and hence k should be

large. However, the starting torque is inversely proportional to k i.e. $T_s \propto 1/k \propto c^2$. Hence, in practice, a compromise is effected and the value of k is about 2.

CAPACITOR STARTING COMPARED WITH SPLIT PHASE STARTING:

A capacitor start motor develops considerably more locked-rotor torque per ampere of line current than the split phase motor. The locked-rotor torque of a single-phase induction motor with two windings displaced 90 deg. is proportional to the product of the following three factors.

1. The sine of the angle of phase displacement between the currents in the two windings.

2. The product of the main winding current multiplied by the auxiliary winding current.

3. The number of turns in the auxiliary winding.

Each of these 3 factors is more favourable in the capacitor start motor.

1. Due to the presence of capacitor, the angle of displacement between the currents in the two windings is greater in the capacitor start motor than in the split phase motor.

2. Since in the capacitor-start motor, the currents in the two windings are wide apart in phase the line current is less than the numerical sum of the currents whereas in the split phase motor, the line current is nearly equal to the numerical sum of currents because of small phase angle

displacement between them. Hence for the same value of line current, more currents can be allowed in the two windings for the capacitor start motor than for the split phase motor and hence more starting torque, for the same line current.

3. The number of turns of auxiliary winding of a split phase motor is limited by the limitation that is necessary on the leakage reactance (which varies with the square of No. of turns) to keep the current in the auxiliary winding to be as nearly in phase with the voltage as possible. There is no such limitation in the capacitor motor, as the reactance of the auxiliary winding is more than neutralised by the capacitor so that more turns can be used in its auxiliary winding. Hence more torque can be developed.

It may not be out of place to mention now the advantage of the capacitor motor (this has been defined in the chapter 1 to be the motor with its auxiliary winding connected in series with the capacitor being across the supply even when the motor is running) which is over looked usually. If we consider the <u>entire</u> cost of a motor installation the capacitor motor is probably cheaper than the usual repulsion induction motor. The entire cost includes the transmission lines, transformers, generators, switches and local wiring. Each of these elements costs less for condenser motors. It had been indicated that in USA a power company can save from $ 10 to $ 20 per motor if capacitor motors were used instead of the usual type. This saving will more than make

up the difference in price between the capacitor motor and other types of motor. If some practical way in which this saving could be passed over to the consumer then the capacitor motor would immediately dominate the field in many applications. This aspect may be noted by various Electricity Boards in India as large number of single phase motors are expected to be installed as the country develops industrially.

PURPOSE OF STARTING SWITCH:

Speed torque curve of a capacitor start motor is shown in Fig. 9. Also a curve connecting the capacitor voltage and rpm. is given, from which it is clear that the capacitor should be cut out before the machine attains speed at which the voltage across capacitor rises high, in order to prevent the burn out of the capacitor. The other reasons for the starting switch is (2) to improve the torque characteristics at full load speed (3) to keep down watts input and (4) to prevent burn outs at normal operating speeds.

Further switches must not be allowed to flutter; for suppose the switch is fluttering and that it interupts the circuit at such a time to leave the capacitor fully charged; then suppose the switch recloses when the voltage is of the opposite polarity - double voltage will be impressed momentarily upon the capacitor which may damage the capacitor seriously.

PREVENTION OF RECLOSING OF STARTING SWITCH:

It has been explained and checked by practical tests by Mr. Veinott that the reclosing of the starting switch if its occurs at synchronous speed can be disastrous to the machine and that this rarely happens at synchronous speed. To understand this, the build up of voltage in an induction machine across the terminals of which are connected capacitances as shown in sketch (No. 10) is to be studied first.

The build up of voltage of the d.c. shunt generator is known to depend upon residual magnetism in the field poles of the machine and upon the resistance of the field circuit.

It has been discovered and shown by E.D. Basset & F.M. Patter that the induction generator with static capacitance connected in shunt across its terminals will build up its voltage in a similar manner. Residual magnetism in the iron of the magnetic circuit sets up a small alternating voltage in the stator; this voltage applied to the capacitance causes a lagging magnetising current to flow in the stator windings (Machine supplies leading quadrature current to the capacitance or draws a lagging quadrature current). If the capacitance is of the proper value, the current that can flow will be large enough to increase the flux existing in the air gap. This will result in a further increase in voltage and so on until its final built up value determined by the saturation curve of the machine and by the capacitive reactance of the connected capacitance, as shown in figure.

A ¼ h.p. capacitor start motor was selected by Mr. Veinott and no load saturation curve was taken (by the method Basset & Patter have given in their paper - by connecting different values of capacitance across the motor and driving the motor). The curve obtained is similar to those shown by Basset & Patter.

In an actual motor of capacitor start type the capacitor is not connected directly across the main winding but may be considered as being shunted across the 2 windings in series - it was assumed that these two windings can be though of as a single winding having a number of turns equal to $\sqrt{N_m^2 + N_s^2}$. This has been checked like this: Short-circuiting the starting switch and inserting various values of capacitance for that shown in the figure. To obtain the magnetisation curve, the voltage is divided by $\frac{\sqrt{N_m^2 + N_s^2}}{N_m}$ and the current multiplied by $\frac{\sqrt{N_m^2 + N_s^2}}{N_m}$ and these values plotted, this new magnetisation curve nearly coincided with that drawn with the main winding and different values of capacitance, thus proving the assumption that the two windings may be considered as a single winding, as shown in figure 12.

It was particularly interesting to note that values of less than half the normal capacitance used with this motor would cause the motor to build up to such high voltages that the power losses were 2 to 3 times the motor rating. If the starting switch of this motor had been short circuited with its normal capacitance connected across the windings and the motor driven to synchronous speed, the motor would have quickly burnt up.

However, this situation is not nearly so serious as it seems and very rarely causes trouble. The starting switch normally does not reclose until the motor has slowed down to approximately half the speed where the conditions are much different since with the machine at such speed, the saturation curve was as shown in c (fig. 13) and the voltage is approximately half that at full speed. However the capacitance required for the machine to build itself up to any given current is 4 times as great as that required at full speed because not only is the voltage halved by reduction in speed but also the frequency. Below a critical value of capacitance the machine will not build up as a generator. This critical value is inversely proportional to the square of the speed.

<u>REFERENCES:</u> B1, B3, B6, B7, J6, J7, J8, J9 & J10.

CHAPTER NO. 3

HARMONICS IN A SINGLE PHASE MOTOR

It has been well established now that many of the ills of the induction motor are due to the harmonics present in the flux wave of the motor. On the fundamental field, myriad of harmonic fields are impressed. The fundamental field is the cause of the flow of energy from the stator to rotor by electro-magnetic induction. The harmonics cause undesirable effects, which are explained in this chapter. The methods used to suppress these harmonics are also described in brief.

Let us first see what are all the harmonics present in a single phase machine and then consider their effects. How these harmonics can be suppressed is discussed in the last pages of this chapter.

The specific problem of noise, though one of the main reasons for this is the presence of harmonics, is however dealt with in a separate chapter.

(a) M.M.F. and field of a single-phase a.c. winding.

Consider a single-phase full pitch winding having 1 slot per pole or per phase of a full pitch winding having 1 slot per phase per pole. The direction of currents in the conductors are as shown in figure 3.1 and also the lines of force with its direction marked. No lines of force are possible which are linked with two coils at the same time. The mmf. of all lines of force is constant and has a maximum value of $\sqrt{2}\, IN_c$

where I is the rms. value of current, N_c the number of turns with which the lines of force are linked. Hence the MMF. distribution at the instant when the current is a maximum (shown in the figure) is a rectangle with its height $\frac{\sqrt{2}}{2} IN_c$.

Now reduce this rectangular wave into its fundamental and harmonics as shown below:

Taking the coordinate axis through the centre of the pole, $f(x) = -f(x)$ hence only cosine terms are present. $f(\pi + x) = -f(x)$; hence only odd terms are present:

$$\therefore \quad f(x) = \sum_{n=1}^{n=\infty} a_n \cos \frac{\pi}{T} x \ n \quad \text{where } n \text{ is odd.}$$

$$\text{But} \quad a_n = 1/T \int_0^{2T} f(x) \cos n \frac{\pi}{T} x \, dx$$

From figure, when $x = 0$ to $x = T/2$, $f(x) = \frac{\sqrt{2}}{2} IN_c \sin wt$

when $x = T/2$ to $x = 3T/2$, $f(x) = \frac{-\sqrt{2}}{2} IN_c \sin wt.$

when $x = 3T/2$ to $x = 2T$, $f(x) = \frac{\sqrt{2}}{2} IN_c \sin wt.$

$$\therefore \quad a_n = \frac{1}{T} \left[\int_0^{T/2} \frac{\sqrt{2}}{2} IN_c \sin wt \cos \frac{\pi}{T} nx \, dx - \int_{T/2}^{3T/2} \frac{\sqrt{2}}{2} IN_c \sin wt. \cos \frac{\pi}{T} nx \, dx + \int_{3T/2}^{2T} \frac{\sqrt{2}}{2} IN_c \sin wt \cos \frac{\pi}{T} nx \, dx \right]$$

$$= \frac{1}{T}\left[\frac{\sqrt{2}}{2} IN_c \sin wt \left(\frac{T}{\pi n}\right)\right]\left[\sin\frac{\pi}{T} n \frac{T}{2} - \sin\frac{\pi n}{T}\frac{3T}{2}\right.$$

$$\left. + \sin\frac{\pi}{T} n \frac{T}{2} + \sin\frac{\pi}{T} n\, 2T - \sin\frac{\pi n}{T}\frac{3T}{2}\right]$$

$$= \frac{1}{T}\frac{\sqrt{2}}{2} IN_c \sin wt \frac{T}{\pi n}\left[2\sin\frac{n\pi}{2} + \sin 2\pi n - 2\sin\frac{3n\pi}{2}\right]$$

$$= \frac{1}{T}\frac{\sqrt{2}}{2} IN_c \sin wt \frac{T}{\pi n}\, 4\sin\frac{n\pi}{2}$$

since, $\sin 2\pi n = 0$ and $\sin 3\frac{n\pi}{2} = -\sin n\pi/2$

$$\therefore\ a_n = \frac{\sqrt{2}}{2} IN_c \sin wt \frac{4}{\pi n}\sin n\frac{\pi}{2}$$

$$= \frac{4\sqrt{2}}{\pi\ 2} IN_c \sin wt \frac{1}{\pi n}\sin\frac{n\pi}{2}$$

$$= .\cancel{0}9\ IN_c \sin wt\ \frac{1}{n}\sin\frac{n\pi}{2}$$

But $f(x) = \sum_{n=1}^{n=\infty} a_n \cos\frac{\pi}{T} nx$

$$\therefore\ f(x) = .9\ IN_c \sin wt \sum_{n=1}^{n=\infty}\frac{1}{n}\sin\frac{n}{2}\cos\frac{\pi}{T} nx.$$

Since $\sin\frac{n}{2} = \pm 1$, for odd values of n

$$f(x) = .9\, IN_c \sin wt \left[\cos \frac{\pi}{T} x - \frac{1}{3} \cos \frac{3\pi}{T} x + \frac{1}{5} \cos \frac{5\pi}{T} x \quad \ldots \right]$$

The rectangular wave, its fundamental and the first two harmonics (3rd & 5th) are shown in figure 3.2.

Thus the amplitude of the nth harmonic is $\frac{1}{n}$ times the amplitude of the fundamental. The pole pitch of the nth harmonic is $\frac{1}{n}$ times the pole pitch of the fundamental.

Even harmonics do not exist, as for even value of n, $\sin \frac{\pi n}{2} = 0$.

In deriving the above equations a single coil has been considered. This is the same as a single-phase winding having 1 slot per pole. If, as is usually the case, q slots per pole are used the mmfs. of the q coils are displaced in phase from each other. Therefore, the distribution factor k_{dn} must be introduced. (The value for this factor is given in the last section of this chapter.).

The mmf. of a single phase winding accordinly is:

$$f(x) = .9\, IN_c\, q \sin wt \sum_{n=1}^{n=\infty} \frac{k_{dn}}{n} \cos n \frac{\pi}{T} x$$

Thus we can see that mmf. of a single phase winding with a full pitch coil has only odd harmonics.

In the concentric winding which is adopted for the

construction of the machine undertaken, the negative half of the mmf. wave is a mirror image of the positive half wave with respect to the axis of the abscissa i.e. $f(x+\pi) = -f(x)$. Therefore, only odd harmonics appear in the mmf. wave.

It must also be seen that the harmonics in consideration are only SPACE HARMONICS and that all space harmonics are produced by the same current, whose time wave form is a pure sinusoid, i.e. the time harmonics are completely absent.

It has been shown that a single phase winding produces an mmf. distribution containing odd harmonics. The permeance variations in the air gap due to slot openings, multiplies the production of field harmonics. In order to ascertain this also it is convenient first to express the air gap permeance as the sum of a constant (average) value and a series of sine-wave variations, then to express the mmf. as a series of harmonic fields, and, finally to multiply these series, term by term, to obtain the resultant air gap magnetic field. But usually the air gap is assumed to be uniform and the permeance constant and the field harmonics evaluated.

(b) Effects produced by harmonic fields:

There are four kinds of effects:

1. Asynchronous crawling. Space harmonics of the winding mmf. create revolving fields which induce secondary currents,

and produce torques, similar to those of the fundamental, but have more poles and, therefore, lower synchronous speeds, As the motor accelerates through the synchronous speed of one of these harmonics, the harmonic torque reverses, causing a dip in the resultant motor torque speed curve, thus seriously impairing the motor's starting ability.

At speeds above their respective synchronous values, the forward harmonics produce braking torques, as the backward harmonics do at all forward speeds.

Thus they create stray load losses and increase the motor heating.

2. Locking and synchronous crawling: If any two of the separate harmonic fields have the same number of poles, pulsating torques will be produces as they slip past each other. When their speeds coincide, the two like fields will synchronise, and a corresponding locking or "synchronous crawling". torque will be observed.

3. Magnetic noise and vibration: If two harmonic fields with numbers of poles differing by 2, coexist in the air gap, they will produce unbalanced radial magnetic forces, and consequent radial vibration of the rotor as a whole. By the super-position of rotating magnetic fields of different poles number, symmetrical radial forces of high frequency are produced. These phenomena create stator vibration and magnetic noise - These are explained in detail in the chapter on noise.

4. Voltage ripples: The harmonic fields produced by the stator current induce currents in the rotor which reflect back into the stator additional harmonic fields, giving rise to terminal voltage ripples and to extra core losses. These voltage ripples, in turn, produce high-frequency currents in the supply lines, which may create inductive interference with communication circuits.

(c) Supression of Harmonics:

Thus we see that the harmonic field effects must be kept under control as a sine qua non of good motor performance.

There are several methods employed to eliminate or reduce the induced voltage harmonics (or harmonic fields), the armature conductors may be SKEWED, the coils may be made with FRACTIONAL PITCH and the winding may be DISTRIBUTED in several slots per pole per phase. The actual reduction in the harmonic fields by these three different methods can be represented by 3 different harmonic factors - expressions for which can be derived. The reduction factors and the seat of reduction are shown in the table below:

THE HARMONIC REDUCTION FACTORS

	Reduction factor		Seat of the reduction
Skew factor	$K_{skn} = \dfrac{\sin n \frac{s}{T_s}\frac{\alpha}{2}}{n \frac{s}{T_s}\frac{\alpha}{2}}$	*	In the coil side
Pitch factor	$k_{pn} = \sin np \frac{\pi}{2}$	**	Between coil sides of the coil.
Distribution factors	$k_{dn} = \dfrac{\sin \frac{n\alpha}{2}}{q\sin(n\alpha/q\ \frac{1}{2})}$	***	Between coils of the phase belt.

where

* S = circumferential length of distribution of a conductor

T_S = slot pitch

α = slot angle.

** p = pitch ratio

*** q = number of slots per pole

α = slot angle.

While these reducing factors have different names, they are in fact due to the same essential cause - the arrangement of the winding so that the harmonic voltages produced in different parts are in partial or complete opposition and thus tend to cancel out over the complete circuit. Figure 3.2 shows how this is accomplished in the case of the third harmonic.

If the conductor is skewed or spiralled so that half of it is cutting through a positive loop, and the other half through a negative loop of the harmonic of space distribution, then the voltage induced in the two halves of the conductor are always indirect opposition and therefore completely cancel within the conductor itself. In the case of fractional pitch, both coil sides are cutting through positive loops of flux, but the voltages generated thereby cancel out in the coil - in this case the coil may be either of short or long pitch. The distribution of the winding in more than one slot per pole per phase gives rise to a reduction of the

harmonic voltages between the coils which make up the phase group.

REFERENCES: B5, B8, and B9.

CHAPTER NO. 4

NOISE IN SINGLE PHASE INDUCTION MOTORS

Psychologists have established that noise reduces human efficiency and hence much attention is being paid nowadays to the problem of noise reduction. Much of the noise present in electrical motor is due to poor manufacture. Careless or inaccurate manufacturing methods often show up in some manner as uneven air gaps, loose bearings, brush noise (in commutator machine) or loose rotor bars. However even with good manufacturing process with due attention given to avoid the above defects there is still noise in an electrical motor; this can be due only to electro-magnetic sources. These are explained in detail in this chapter. How this noise can be reduced considerably ? This is dealt with in the latter portion of this chapter.

(A) Radial forces due to the air gap field:

By far the largest sources of magnetic vibration and noise in electrical machines are the radial forces due to the air gap field. If the flux were sinusoidally distributed, the magnetic forces around the periphery would be a simple $\sin^2 wt$. curve that is a fully displaced sinusoid with twice as many poles as there are magnetic poles. Actually as previously shown in the chapter on Harmonics there are a numerous train of harmonic fields superposed on the fundamental

flux wave giving rise to high frequency pulsations in the radial magnetic forces.

The total radial forces are proportional to the (flux density)^2 which is the (sum of all stator and rotor harmonics)^2 Normally the radial forces produced by the (stator harmonic flux density)^2 alone are not disturbing since their frequency is either low (twice line frequency) or high (approximately twice slot frequency). The forces due to the (2 stator harmonic x rotor harmonic) may be very disturbing. Such an expression for radial force (obtained by the product of a stator harmonic with a rotor harmonic) can be resolved into two travelling force waves, one having a lower number of pole pairs than the other. The larger number of force pole-pairs lead to short wave distortion to which the stator is stiff. The lower number of force pole pairs lead to long-wave periodic distortion of the stator with time. The external surface of the frame in turn beats the air causing air-borne noise.

In what follows, the harmonic theory of noise is explained in detail, concentrating the attention only on the tooth harmonics. This theory helps in the evaluation of noise level in a quantitative manner. Then it is shown how a two-pole-difference between two fields existing in the air-gap of a machine results in an unbalanced rotating force that causes much vibration and noise; this has been taken as of some special interest since such combination of two fields renders the motor so noisy as to make it practically useless; further the slot combination (which is

discussed later) is made with one of its object being to avoid such fields. A method published by Veinott to predict the noise frequencies that may be expected in a machine is described: though this method is of no use in determining noise quantitatively, its simplicity has been the main reason for its finding a place in this dissertation. The noise frequencies that may be present in our motor are calculated by this method. Lastly latest work on noise produced by the dissymmetries in the machine is just mentioned.

(a) J. Morrill's development of harmonic theory - the part played by tooth harmonics in the production of noise:

With symmetrical windings harmonics fluxes, may exist having a number of poles corresponding to any odd multiple of the fundamental number of poles. Good winding distributions generally limit the magnitude of harmonics of low order; but with a given number of stator slots uniformly spaced, nothing can be done to reduce the magnetudes of the tooth harmonics as the distribution factor for the tooth harmonics is identical with that for the fundamental flux.

There always exist an infinite series of tooth harmonics having magnitudes and orders as shown below:

$$\text{Order of harmonic (tooth)} = n_{s\ell} = \pm \frac{k_1 Q_1}{p/2} + 1$$

k_1 = any integer

Q_1 = Total No. of teeth.

p = total no. of poles.

$$\text{Amplitude of Harmonic wave} = \frac{A_1}{n_{s\ell}}$$

where A_1 = amplitude of fundamental flux wave.

It can be noticed that the tooth harmonics go in pairs and that if there are nine teeth per pole in a 4 pole motor the largest pair of tooth harmonics are the 17th and 19th harmonics. The next largest harmonics will be the 35th and 37th.

In the figure No. 4-1, the stair-step wave of flux produced by a four-pole winding in a 36 slot stator, with the fundamental and the first pair of tooth harmonics corresponding to the stair-step wave, superposed on it, is shown. Both the harmonics and the fundamental wave of flux should be visualized as pulsating with time; the effect of the pulsation is exactly duplicated by dividing each pulsating wave into two oppositely gliding components each having half the amplitude shown. If the forward and backward components be used, there are now six stator flux waves to be considered. If two subscripts N & M represent the order of the first pair of tooth harmonics and subscript s represent that associated with the stator, these six waves are:

$$W_{sf} = A_{sf} \cos \left[x \frac{\pi}{\lambda} - wt \right]$$

$$W_{sb} = A_{sb} \cos \left[x \frac{\pi}{\lambda} + wt \right]$$

$$W_{sNf} = A_{Nf} \cos \left[Nx \frac{\pi}{\lambda} - wt \right]$$

$$W_{sNb} = A_{Nb} \cos \left[Nx \frac{\pi}{\lambda} + wt \right]$$

$$W_{sMf} = A_{sb} \cos \left[Mx \frac{\pi}{\lambda} - wt \right]$$

$$W_{sMb} = A_{Mb} \cos \left[Mx \frac{\pi}{\lambda} + wt \right]$$

Thus the above six equations represent the six gliding waves due to the fundamental and the first pair of stator tooth harmonics.

Now the effect of these waves on the rotor will be considered. Only the fundamental gliding components, W and W produce rotor currents which produce wave that play effective part in the performance of the machine are considered. The effect of the gliding waves due to the first pair of tooth harmonics and other harmonics is neglected without much error.

The forward component of fundamental stator flux W_{sf} acts on the rotor windings to produce a forward wave of rotor currents which in turn produces a forward wave of rotor fundamental flux moving with respect to the rotor. Associated with this forward flux is an infinite series of rotor tooth harmonics fluxes. Unlike the stator fluxes, the rotor fluxes, arising as they do from the action of a gliding wave of stator flux having constant amplitude, are not pulsating. They glide forward and backward on the rotor (forward with respect to the stator) with constant magnitude and at slip frequency with respect to the rotor.

The rotor tooth harmonics have poles and amplitudes as shown below:

Order of rotor harmonic $= \pm \dfrac{k_2 Q_2}{p/2} + 1 = \mu_{s\ell}$

Amplitude of rotor harmonics $= \dfrac{A_{rs}}{\mu_{s\ell}}$

Q_2 = total no. of rotors slots;

k_2 = any integer.

A_{rs} = Amplitude of forward fundamental rotor wave.

R will indicate the negatively moving rotor tooth harmonic and the ratio of its poles to the poles of the fundamental and T will indicate the positively moving rotor tooth harmonic and the ratio of its poles to the poles of the fundamental.

Similarly the backward component of fundamental stator flux W_{sb} is responsible for producing a backward component of fundamental rotor flux gliding backward on the rotor at a frequency, $(2 - s)\,f$; corresponding tooth harmonics are also produced.

Now only the fundamental and first pair of tooth harmonics of the rotor be considered, there are in all six rotor fluxes three due to W_{sf} and three due to W_{sb}. Neglecting the difference in time phase between stator and rotor currents, these six fluxes may be expressed with respect to the rotor as shown below:

$$W_{rf} = A_{rf} \cos \left[x_2 \frac{\pi}{\lambda} - wst \right]$$

$$W_{rb} = A_{rb} \cos \left[x_2 \frac{\pi}{\lambda} + w\,(2-s)t\right]$$

$$W_{rRf} = A_{Rf} \cos \left[Rx_2 \frac{\pi}{\lambda} + wst\right]$$

$$W_{rTf} = A_{Tf} \cos \left(Tx_2 \frac{\pi}{\lambda} - wst\right)$$

$$W_{rRb} = A_{Rb} \cos \left[Rx_2 \frac{\pi}{\lambda} - w(2-s)t\right]$$

$$W_{rTb} = A_{Tb} \cos \left[Tx_2 \frac{\pi}{\lambda} + w(2-s)t\right]$$

With respect to the stator the above rotor waves can be written:

$$W_{rf} = A_{rf} \cos \left(x \frac{\pi}{\lambda} - wt\right)$$

$$W_{rb} = A_{rb} \cos \left(x \frac{\pi}{\lambda} + wt\right)$$

$$W_{rRf} = A_{Rf} \cos \left[Rx \frac{\pi}{\lambda} - w\,\overline{R(1-s)-s}\;t\right]$$

$$W_{rRb} = A_{Rb} \cos \left[Rx \frac{\pi}{\lambda} - w\,\overline{R(1-s)+2-s}\;t\right]$$

$$W_{rTf} = A_{Tf} \cos \left[Tx \frac{\pi}{\lambda} - w\,\overline{T(1-s)+s}\;t\right]$$

$$W_{rTb} = A_{Tb} \cos \left[Tx \frac{\pi}{\lambda} - w\,\overline{T(1-s)-2+s}\;t\right]$$

Summarising, first of all we considered the pulsating stator fundamental and first pair of tooth harmonics (of a single phase motor) as giving rise to six stator gliding waves, W_{sf}, W_{sb}, W_{smf}, W_{snb}, W_{snf} and W_{sNb}. Out of these six rotating waves only the effect of the two fundamental components, W_{sf} and W_{sb}, on the rotor was taken into account. Though myriads of waves will be produced by these two waves only the fundamental and the first pair of rotor tooth

harmonics due to each of these waves W_{sf}, and W_{sb}, were then considered, thus leaving only six rotor waves namely, W_{rf}, W_{rb}, W_{rRf}, W_{rRb}, W_{rTf}, and W_{rTb}.

The flux at every point in the air gap of the motor and at every instant of time is exatly representable as the sum of the fundamental fluxes and all the harmonic fluxes 12 waves mentioned in the previous paragraph. Hence using those which have been considered, we may write:

$$B_d = W_{sf} + W_{sb} + W_{sNf} + W_{sNb} + W_{sMf} + W_{sMb} + W_{rf} + W_{rb}$$
$$+ W_{rNf} + W_{rNb} + W_{rMf} + W_{rMb}$$

The radial force density at any point in the gap due to B_d is, according to maxwell:

$$\frac{F}{A} = \frac{1}{72,130,000} B_d^2 \text{ pounds per sq. inch.}$$

For noise study it is desirable to work with the components of the force density, which components are obtainable by the expansion of the square of the expression for B_d. This leads to 78 terms which are clumsy to handle. The expansion consists of 8 squares and terms which are product of each term with each other term. Let us perform one of the multiplication and see. In the case of the N forward stator tooth harmonic and the R forward rotor harmonic, the product is:

$$F\ (N_f)\ (R_f) = 2\ A_{nf}\ A_{Rf} \cos\left(Nx\ \frac{\pi}{\lambda} - wt\right)$$

$$\cdot \cos\left[Rx\ \frac{\pi}{\lambda} - w\ \overline{R\ (1 - s) - s)}t\right]$$

$$= A_{Nf}\ A_{Rf} \left\{ \cos\left[(R - N)\ x\ \frac{\pi}{\lambda} - w\ \overline{R\ (1 - s) - 1 - s}\ t\right]\right)$$

$$+ \cos\left[(R + N)\ x\ \frac{\pi}{\lambda} - w\ \overline{R\ (1 - s)\ \ 1 - s}\ t\right]\left.\right\}$$

Thus it can be seen that the product of the two flux waves is representable as two force waves, the upper wave having a long wave length and the lower one having a very short wave length. Very short force waves will not be important because the stator will be very stiff to short-wave distortion.

Long waves of force, however, will cause a relatively large response and therefore will be important in producing noise.

The pitch of the long force wave is

$$\lambda\ (N_f)\ (R_f) = \frac{\lambda}{R - N}$$

The frequency of this wave is:

$f_{(Nf)\ (Rf)} = f\left[R\ (1 - s) - 1 - s\right]$ and this frequency is the frequency at which the air-borne noise will be emitted.

A study of various products show that the products most likely to be responsible for noise are the ones of long pitch produced by multiplication of a stator harmonic with a rotor harmonic. There are 16 such products. The frequencies

of these sixteen forces responsible for noise are functions of only the slip and the number of rotor slots.

(b) <u>How a difference of two poles between two fields produce unbalanced force:</u>

Now let us see how this difference of two poles between two fields existing in the air gap of the induction machine, cause noise. Let us assume a 4 pole stator producing sinusoidal flux wave ϕ_1. Also assume there are a total of 5 rotor slots. Figure 4.2 shows the relative positions of the rotating field and the rotor conductors at a certain instant. The emfs. are induced in the rotor bars and as they are short-circuited currents will flow through them. The stepped curve (abcdefghkl) in the next figure (No. 4.3) shows the distribution of the field, ϕ_2, due to the rotor currents at the instant when the flux in tooth 1 is at its maximum value. The curve ϕ_3 which bounds the shaded areas is obtained by adding together the ordinates of the curves ϕ_1 & ϕ_2 at each point along the rotor surface. The force with which any element of the rotor surface is attracted towards the stator is proportional to the square of the flux density at that point as already indicated. The lengths of the radial lines drawn in the figure No. 4.4 are proportional to the squares of the resultant densitites (ϕ_3) given in the previous figure. It is clear from this diagram that the rotor experiences a resultant force directed radially outwards through the centre of the slot (3,4).

If the mmf. diagrams are drawn for other instants and the resultant mmf. is squared at each point, to find the force there, it will be found that the resultant force vector makes two revolutions per cycle and therefore rotates 4 times as fast as ϕ_1 relatively to the rotor.

This unbalanced force tends to bend the shaft of the rotor and, under steady conditions, the plane of the neutral axis will rotate at the same speed in space as the unbalanced force. The shaft being bent the air gap is reduced on one side and an increased unbalanced magnetic attraction is produced which is added to the original disturbing force and further since the centre of gravity of the rotor is whirling with the neutral axis in a circle, a centripetal mass-acceleration is required to maintain the motion. Steady motion is obtained when the shaft is bent to such an extent that the elastic forces set up by bending are just sufficient to balance the magnetic attration and to provide the requisite mass-acceleration. Such steady conditions cannot exist while the rotational speed of the rotor is changing, because the frequency of the forces brought into play is changing continuously.

A useful light is thrown on the phenomenon of the production of unbalanced force, if we apply the fourier's analysis to the fields shown in the diagram (No. 4.3). In the figure (No. 4.5) the curve ϕ_3 represents the difference between ϕ_1 and the 4 pole component of ϕ_2 (i.e. fundamental component) a flux wave which moves from right to left like ϕ_1.

ϕ_4 represents the six pole component of ϕ_2 which moves from left to right. ϕ_5 represents the resultant of these two flux waves at a certain instant and the corresponding force diagram is shown in the next diagram (No. 4.6)

These two diagrams show the characteristic feature of the resultant field produced by a p-pole field with a $(p \pm 2)$ pole field, viz., that there is a strong zone the centre of which is immediately opposite to that of a weak zone, which leads to a pronounced unbalanced magnetic pull. This force can be represented by a radial vector rotating about the axis of the rotor.

If a p-pole field be combined with a $(p + x)$ pole field, where $x = 4, 6, 8, 10$ etc., there will be $x/2$ strong and $x/2$ weak zones distributed regularly round the periphery of the rotor and the forces will constitute a balanced system. It is only when $x = 2$ that unbalanced forces arise.

(c) <u>Veinott's "permeance method" to find the noise frequencies that may be expected from a motor:</u>

It can be seen from Morrill's treatment that the noise frequencies that are predominant in the motor depend solely upon the number of rotor teeth and rpm. and not upon the number of stator teeth or upon the combinations.

Consider any single stator tooth carrying flux. As each rotor tooth passes a stator tooth, a pull is exerted on the latter. There are 20 rotor teeth in our machine which

is revolving at 1500 rpm. (synchronous) or 25 v evolutions per sec; then, the frequency of the radial force on the stator tooth is:

20 x 25 = 500 cycles per sec.

If the stator is energised at 50 cycles, the frequency of the force of attraction between the rotor and the stator is 100 cycles. This 100 cycle force is super-imposed upon the 500 cycles giving two side band frequencies either side of 500. The three frequencies are, therefore:

500 - 100	=	400	cycles per sec.
		500	cycles per sec.
500 - 100	=	600	cycles per sec.

The second harmonic of the stator tooth pulsation frequency due tothe rotor teeth and the 100 cycle side band frequencies are:

2 x 20 x 25	=	1000	cycles per sec.
1000 - 100	=	900	cycles per sec.
1000 100	=	1100	cycles per sec.

The third harmonic and accompanying side band frequencies are:

2 x 20 x 25	=	1500	cycles per sec.
1500 - 100	=	1400	cycles per sec.
1500 100	=	1600	cycles per sec.

Pursuing this method, we might expect to find other frequencies present which are function of the number of stator teeth.

The pulsation frequency in the rotor teeth at 1800 rpm. with 28 stator teeth is:

28 x 25 = 700 cycles per sec.

We might logically expect a pull between the stator and rotor at frequencies of:

700 cycles per sec.
700 - 100 = 600 cycles per sec.
700 100 = 800 cycles per sec.

We might also expect to find frequencies of 2 x 28 x 25 = 1400 cycles per sec.

1400 - 100 = 1300 cycles per sec.
1400 100 = 1500 cycles per sec.

The above method is quite useful to predict the noise frequencies that may be expected. But to find the possible noise frequencies are not of much help to predetermine the noise of a motor. To determine the actual harmonic forces Marrill's method outlined already is quite useful.

(d) <u>Noise due to dissymmetry harmonics</u>:

All the theories till now considered, is defined by D.F. Muster and G.L. Wolfert as " the first-order motor

noise problem". This includes proper selection of such items as a rotor stator slot, combination, winding distribution, amount of skew, open or closed slots etc. so that the noise level expected from a specific motor design will be as low as possible. It is assumed in these theories that the permeability of the rotor and stator iron is infinite (no saturation exists), the stator voltage and current are sinusoidal and the air-gap is uniform around the periphery of the motor. A motor designed in accordance with these theories should produce a minimum of magnetic noise with components at only 100 cycles per second and the frequencies of the basic slot permeance harmonics. Qualitative treatment of this has been done by many some of which have been described. Among the quantitative treatment Morrill's analysis has been dealt with.

However, the motors still produce noise much of which cannot be accounted for by theory. This is due to the effect of dissymmetries (mechanical and electromagnetic), nonlinearities (e.g. caused by saturation) non uniformities and the inevitable clearances and tolerances incident to the mass production of motors. In their paper Muster and Wolfert have presented a method of (qualitative) analysis which yields the characteristics of radial force waves produced in a single phase induction motor by intermodulation between air gap permeance variations caused by rotor and stator slots and rotor and stator dissymmetrics, winding distribution harmonics and stator current harmonics.

This method is not elaborated here. Mr. P.L. Alger, describes this method as an important progress towards the right direction in the analysis of noise in rotating, machinery and hopes that this will be followed by quantitative analysis of the "residual problem" (the noise that is not explained by the first order motor noise analysis).

B. <u>Other reasons for noise in a single phase machine</u>:

<u>Magnetostriction:</u>

A magnetic material, when magnetised changes its dimensions. This is called magnetostriction. Most of the steels used in electrical machines lengthen parallel to the direction of the flux lines and contract at right angles when the intensity of magnetisation is increased. This is independent of the polarity and hence, an alternating flux causes the core surface to pulsate at twice the supply frequency. Though the variations in the dimensions are small it is enough to set up sound waves in the surrounding medium.

C. <u>Coil vibration:</u>

When conductors carry currents forces are set up which cause vibration of the conductors. These forces are proportional to the square of the current and the vibrations will be at double frequency.

D. <u>Inter laminar fluxes:</u> may also result in vibration of the individual laminations and this may be source of

noise if the core laminations are not tightly held together.

E. Resonating parts: in the machine and fan intensifies, noise.

F. Single-phase torque pulsation:

The inherent pulsating nature of the single phase power yields a pulsating torque component in the output. This causes further noise in a single phase motor. The pulsation of torque has been dealt with extensively in another chapter.

NOISE REDUCTION METHOD.

The main causes for the noise in a single phase motor having been discussed at length let us proceed to find the methods to produce quiet motors.

It has been seen that field harmonics produce unbalanced radial force around the periphery of the induction motor which causes vibration and noise. Harmonic reduction and the factors for such reduction by various methods have been explained in another chapter briefly. Larger air-gap reduce harmonics to a certain extent. Elastic mounting suppresses the vibration & noise due to the double-frequency torque inherent in a single phase motor. These methods are touched briefly.

Except skewing other methods of harmonic reduction (distribution of winding in a number of slots, having short or long pitch coils) cannot reduce the tooth harmonics and

so these are the most disturbing fields as far as noise is concerned. Since the tooth harmonic fields depend upon the numbers of slots in the stator and rotor, these numbers and their combination are important. The slot combination is discussed in detail, starting with a method to find a suitable number of rotor slots which will avoid fields with two pole difference. Then Kron's rules to avoid lower number of force poles are stated in brief. Lastly Garik's method of Harmonic Chart is extended to the single phase motor by charting out all the harmonics that may be present in our motor and checking up for lower number of force pole-pairs that may be present.

1. Slot Combination:

It has been seen that the existence of lower number of force poles due to the harmonic fields was one of the main reasons for noise. Since harmonics other than tooth harmonics can be reduced to a considerable extent by methods explained in another chapter, only tooth harmonics could play an important part in the production of noise. In order to avoid the presence of lower order of force poles suitable slot combinations may be adopted, so that the harmonic fields introduced by slots would not produce lower order of force poles. Let us first consider a rule determined by Chapman which enables us to avoid 2 poles difference between two fields which results in unbalanced rotating force as already seen.

(a) Chapman's rule to choose numbers of slots for squirrel cage rotors which will avoid fields differing by 2 poles:

The stator of a single phase field will produce harmonics which can be represented by:

$n = 4k_1 \pm 1$ where k_1 is any a positive integer. Hence with p poles, the possible numbers of poles in the component stator fields are p, $3p$, $5p$, $7p$ etc., or generally $(4k_1 \pm 1)p$. If Q_2 be the No. of rotor slots then the possible values of rotor harmonics due to the nth harmonic form an infinite series and the general term is:

$$= \frac{Q_2k_2}{p/2} + n \quad \text{where } k_2 \text{ is any positive or -ve integer.}$$

$$= \frac{2Q_2k_2}{p} + n$$

$$\therefore \quad p\mu = 2Q_2k_2 + np$$

$\therefore$ For a single phase, we must have

$$(4k_1 - 1)\,p \pm 2 \neq 2Q_2k_2 + (4k_3 \pm 1)p$$

where k_1 & k_3 are positive integers.

$$4p(k_1 - k_3) \pm 2 = 2Q_2k_2$$

$$\therefore \quad Q_2k_2 \neq pk_4 \pm 1 \quad \text{where } k_4 \text{ is a positive or negative integer.}$$

Since $k_2 = 1$ and $k_2 = -1$ are the only, cases of practical importance we finally obtain.

$Q_2 \neq pk_4 \pm 1$ <u>i.e. the number of rotor slots must not differ by unity from any multiple of the number of poles for which the stator is wound.</u>

In the case of a 4 pole machine, for instance, the rotor must not have an odd number of slots.

(b) <u>Kron's rules:</u>

More recent investigation and articles have added still more to our fund of knowledge on the selection of the number of rotor bars which will let us design the motor still quicker and eliminate some that would still retain if we tried all those without a 2-pole difference.

Mr. Gabriel Kron states that vibration and noise are liable to be present as follows:

1. When the slots differ by half the nuber of poles torsional vibration and noise may occur.

2. When the slots differ by one or by the number of poles plus or minus one, transverse vibration and noise may occur.

3. When the slots differ by the number of poles, torsional vibration and noise may occur and also rumbling noise unaccompanied by critical vibrations.

The chance that the noise occurs in the working range is great with smaller number of poles, higher speed, and with small

transverse or torsional critical speeds. When these noises do occur the motor is practically useless.

(c) Garik's harmonic chart to determine suitable slot combination - modified to suit single phase motors:

All the harmonics that are present in the flux of the air gap of the motor can predicted, by the method of harmonic chart, explained by Mr. Garik, modified suitably for the single phase condition. This chart can easily be used to verify whether the slot combination selected will produce a quiet motor or not. The method is a very simple one.

The current in the stator of a single phase motor produce pulsathy space harmonics which can be represented by

$$n = 4 k_1 \pm 1 \quad \text{where } k_1 \text{ is any positive integer.}$$

This represents only pulsating harmonics, being single phase. A pulsating field can be resolved into two oppositely revolving fields each having $\frac{1}{2}$ the amplitude as the pulsating field; these revolving harmonics can then be represented by

$$n = \pm(4k_1 \pm 1) \quad \text{where } k_1 \text{ is any positive integer.}$$

Each one of these revolving harmonics induce rotor flux which can be represented by an infinite series, the general term of which is:

$$= \frac{k_2 Q_2}{p/2} + n \quad \text{where } k_2 \text{ is any positive or negative}$$

integer including zero.

The stator slot harmonics $= n_{s\ell} = \pm \dfrac{Q_1}{p/2} + 1$

where Q_1 = total no. of stator slots.

The rotor slot harmonics $= \mu_{s\ell} = \pm \dfrac{Q_2}{p/2} + 1$

In order to avoid the inconvenience of calculation with fractional harmonics, it is expedient to introduce as reference wave, a two pole wave, the length of which is equal to the circumference of the machine, regardless of whether such a harmonic exists or not.

Hence as per this 2-pole reference wave, the main wave will now be the p/2th harmoni-c. Hence the formula given before for the various harmonics are each to be multiplied by p/2 in order to get their order in terms of the new reference wave.

$$n' = \pm (4k_1 \pm 1)\, p/2 \qquad \mu' = (k_2 \frac{Q_2}{p/2} + n)\, p/2$$

$$= (k_2 Q_2 + n')$$

$$n'_s = (\pm \frac{Q_1}{p/2} + 1)\, p/2 \qquad \mu'_{s\ell} = (\pm \frac{Q_2}{p/2} + 1)\, p/2$$

The actual harmonic chart is given for the machine constructed for this thesis. The number of stator and rotor slots are 28 and 20 respectively.

$\therefore \; Q_1 = 28 \; ; \; Q_2 = 20.$

Stator tooth harmonics $= \pm \dfrac{28}{4/2} + 1 \quad = \pm 14 + 1$

$= 15 \ \& \ -13$

Rotor tooth harmonics $= \pm \dfrac{20}{4/2} + 1$

$= \pm 10 + 1 \quad = 11 \ \& \ -9$

Refering this to the new reference, the

$n'_{s\ell} = 30 \ \& \ -26.$; $\mu'_{s\ell} = 22 \ \& \ -18$

With respect to noise, the most disturbing combinations of flux waves (of stator and rotor) are (a) those of the stator slot harmonics with the rotor slot harmonics and (b) of the rotor harmonics produced by the stator slot harmonics with the main wane of the stator, For these combinations low values of force pole pain are to be avoided. In the present case, the force pole pairs due to the combination referred under (a),is

$p'_1 = -26 + 22 = 4$ force pole pairs.

and due to the combination referred under (b) is:

$p'_2 = 2 - 6 = 4$ (4 force pole pairs).

Hence in this motor the minimum force pole pairs is 4. $p' = 1$ is the most disturbing one. This is absent in this motor. Therefore the motor should not have much vibration and noise.

k_1	Stator harmonics $n' = \pm(4k_1 \pm 1)p/2$	μ' = Rotor harmonics = $k_2Q_2 + n'$		
		$k_2 = 0$	$k_2 = 1$	$k_2 = -1$
0	2 main wave	2	22	-18 μ_{SR}
	-2	-2	18	-22
1	6	6	26	-14
	-6	-6	14	-26
	10	10	30	-10
	-10	-10	10	-30
2	14	14	34	-6
	-14	-14	6	-34
	18	18	38	-2
	-18	-18	2	-38
3	22	22	42	2
	-22	-22	-2	-42
	+26	26	46	6
	-26 n'_{sl}	-26	-6	-46 *
4	30	30	50	10
	-30	-30	-10	-50
	34	34	54	14
	-34	-34	-14	-54
5	38	38	58	18
	-38	-38	-18	-58
	42	42	62	22
	-42	-42	-22	-62
6	46	46	66	26
	-46	-46	-26	-66
	50	50	70	30
	-50	-50	-30	-70

* rotor harmonics produced by stator slot harmonics.

2. <u>Harmonic reductions:</u>

As the harmonic fields produce noise as indicated in the earlier portion of this chapter it is clear that these fields should be minimised. In the chapter on harmonics different methods have been indicated to reduce the harmonic fields. The slot harmonics produce objectionable noise frequencies very near the peak sensitivity of human ear. As only skewing the rotor slots or the stator slots is the only method to reduce these harmonics, skewing is an important method to reduce noise in motors.

3. <u>Production of sinusoidal wave form:</u>

If we make the air gap wave form as sinusoidal as possible the harmonic contents of the space flux is reduced. Khulman gives in his book on the Design of Electrical Machines, a method to determine the winding in order to have a sinusoidal distribution of flux. This method is adopted in the design of the single phase motor made for this thesis and the design chapter may be referred to for this method.

4. <u>Larger air gap:</u>

Since large air gap lowers the field harmonics, an increased in the gap width therefore reduces the noise. However larger air gap is not permissible in small induction motors as this would further worsen its already poor power factor.

The lamination, shaft and frame, should be made adequately stiff and the natural frequencies of the mechanical parts should not coincide with the frequencies of the unpressed magnetic forces.

Alger writes that care should be taken to avoid mechanical resonance of all parts of the stator frame and end shields in the frequency band $(\frac{Q_2}{p} \pm 2)$ line frequency as otherwise objectionable noise might be produced. For the machine constructed for this thesis, $Q_2 = 20$; $p = 2$ Therefore the frequency band is 400 - 600 cycles per sec. This is the band of frequencies of the radial force due to the rotor teeth as found earlier by Veinott's "Permeance method" (to find the noise frequencies in an induction motor.

5. <u>Elastic mounting:</u>

The pulsating torque of a single phase motor is also a source of noise as already mentioned. One remedy is an elastic mounting such as a rubber ring around the bearing housing, providing torsional elasticity with a maximum of radial stiffness.

<u>REFERENCES</u>: B5, B8, J11, J12, J13, J14, J15, J16 and J17.

CHAPTER NO. 5

DOUBLE REVOLVING FIELD THEORY

The revolving field theory is first applied to a purely single phase operation, i.e., with only one winding connected to the alternating current supply. The fundamental concept which is the basis of the revolving field theory is explained; the motor is considered as equivalent to 2 two-phase motors and based on this equivalent circuit is evolved and basic equations for current, torque, output etc. are derived.

Then the single-phase motor with both the main and starting windings across supply is considered. Mr. Wayne J. Morrill's classic paper on capacitor motor is mainly followed to draw equivalent circuit and to obtain expressions for currents, torque (starting and running), output etc.

Lastly the method of performance calculation based on the double revolving field theory - called "Apparent Impedance Method" - is explained in detail.

I. Consideration with only the main winding connected to supply.

(a) Fundamental concept.

This theory makes use of the fact that a pulsating

field is equivalent to two equal and oppositely rotating fields, each of which develops its own torque on the motor. At standstill the two torques are equal and opposite; but for any rotor speed, one of the torque dominates, because the slips are different.

Assume the single-phase stator flux to be sinusoidally distributed in space and varying sinusoidally in time. If 2T be the wave length of the space distribution, the flux may be represented by

$$\phi = \phi \sin \frac{\pi x}{T} \cos wt \qquad (1)$$

$$= \frac{\phi}{2} \sin \left(\frac{\pi x}{T} + wt \right) + \sin \left(\frac{\pi x}{T} - wt \right)$$

Therefore $$\phi = \frac{\phi}{2} \sin \left(\frac{\pi x}{T} - wt \right) + \frac{\phi}{2} \sin \left(\frac{\pi x}{T} + wt \right) \qquad (2)$$

= (Forward wave) + (Backward wave)

That any function $f\left(\frac{x}{T} \pm wt \right)$ is a travelling wave of fixed shape and magnitude can be checked as below:

Let us put $\frac{x}{T} \pm wt$ = Constant = k. Then this defines (3) a particular value of, or point on, the given function. For any instant of time "t" there is a corresponding "x" for which the function has this same value of constant k.

In fact the velocity with which the point travels is found by differentiating the above expression.

$$\frac{\pi x}{T} \pm wt = k \tag{4}$$

$$\frac{\pi}{T}\frac{dx}{dt} \pm w = 0 \qquad \therefore \frac{dx}{dt} = \pm \frac{wT}{\pi} \tag{5}$$

Thus the pulsating flux is expressible as a pair of oppositely rotating fields of equal magnitude (See fig. 5.1)

The exposition sometimes found in text books of putting $\cos wt = \frac{e^{jwt}}{2} + \frac{e^{-jwt}}{2}$ and concluding that the pulsating field has thereby been expressed as a pair of oppositely rotating fields in hardly acceptable, since the motion of a travelling wave or rotating field connotes something happening <u>in space and in time</u>. There is nothing about space in e^{jwt}; it refers only to a time diagram.

If the motor speed is w_o and synchronous speed $= w$, then the slips with respect to the forward and backward fields are:

$$s = \frac{w - w_o}{w} = 1 - \frac{w_o}{w} = 1 - \frac{f_o}{f} \tag{6}$$

$$s_b = \frac{w - (-w_o)}{w} = 1 + \frac{w_o}{w} = 1 + \frac{f_o}{f} = 2 - s \tag{7}$$

(b) <u>Single phase motor equivalent to two 2 phase motors</u>.

It is convenient to consider the single phase motor as two superimposed 2-phase motors, having forward and backward

rotating fields respectively as shown in the figure 2. The actual stator winding of N_m turns contributes $N_m/2$ turns to each 2-phase motor and the stator currents are the same in each so that the phase-1 ampere turns balance is,

$$\frac{N_m I_m}{2} + \frac{N_m I_m}{2} = N_m I_m \qquad (8)$$

But the currents in the phase 2 coils must be opposed in order to develop revolving fields of opposite direction and their superimposed ampere turns therefore cancel. Thus the two fictitious polyphase machines superimpose to form the actual single-phase motor.

Each 2 phase motor has a stator resistance of R_s ohms per phase and reactance of X_s ohms per phase. Since the stators are in series there must be $2 R_s = r_s$ and $2X_s = x_s$ to satisfy the single-phase stator impedance. Therefore $z_{sm} = 2Z_s$

(c) <u>General equations from the revolving field theory:</u>

Refering to figure 5.2 and using subscripts f and b to distinguish the forward and backward field, we can write the general equation for the motor.

The stator voltage equation is

$$V_m = V_{mf} + V_{mb} = Z_s I_{mf} + E_{fm} + Z_s I_{mb} + E_{bm}$$

$$= 2Z_s I_m + E_{fm} + E_{bm} \quad (\text{since } I_{mb} = I_{mf})$$

$$V_m = z_{sm} I_m \quad E_{fm} \quad E_{bm} \qquad (9)$$

The two rotor voltage equations, referred to the stator are,

$$0 = E_{fm} + \left(\frac{r_r}{2s} + j \frac{x_r}{2} \right) I_{rf'} \qquad (10)$$

$$0 = E_{bm} + \left(\frac{r_{r'}}{2(2-s)} + j \frac{x_r}{2} \right) I_{rb'} \qquad (11)$$

In the two equations above a distinction has been made between the referred rotor resistance for the forward field $\frac{r_r}{2}$ and that for the backward field $\frac{r_{r'}}{2}$. In the former case the rotor frequency is very small and so the skin effect is negligible, while in the latter case the frequency is about twice the stator frequency and the skin effect very high.

For each polyphase motor the sum of the stator and rotor currents (referred) is the exciting current responsible for the main flux and for the careless, as accounted for by the exciting impedances.

$$I_{\phi b} = I_m + I_{rf'} = \frac{E_{fm}}{z_{m/2}} \qquad (12)$$

$$I_{\phi b} = I_m + I_{rb'} = \frac{E_{fm}}{z_{m/2}} \qquad (13)$$

(d) The equivalent circuit, torque and output :

The above five equations, N_o. 9 to No. 13 can be interpreted in terms of the equivalent circuit shown in the figure No. 5.3

On blocked rotor rest, s = 1, the impedance through the rotor circuits is small compared with the exciting impedances, and the circuit reduces approximately to fig. 4, and

$$V_m \simeq \left[r_{sm} + jx_{sm} + \tfrac{1}{2}\,(r_r + r_{r'}) + j\,x_r\right] I_m \qquad (14)$$

from which it is clear that the measured test values of the rotor impedance are double the equivalent circuit values.

On running light test, s = 0, the impedance branch through the $r_r/2$ branch is infinite and that part of the circuit is open, but the impedance through the $r_{r'}/2$ branch is small compared to the exciting impedance, fig.5; and so.

$$v_m = \left(r_s + jx_s + \frac{r_m}{2} + j\frac{x_m}{2} + \frac{r_{r'}}{4} + \frac{jx_r}{2}\right) I_m$$

$$\therefore V_m \simeq \left(\frac{r_m}{2} + \frac{jx_m}{2}\right) I_m \qquad (15)$$

Hence $\frac{z_m}{2}$ can be determined from volt, current and power measurements under running light conditions. Solution of the equivalent circuit, in fig. 5.3, for the currents I_{rf} and I_{rb} giyes the power input to the rotor, as,

$$P_o = \frac{I_{rf}^2\, r_r}{2s} + \frac{I_{rb}^2\, r_{r'}}{2(2-s)} \qquad (16)$$

Subtracting the copper losses $I_{rf}^2 \dfrac{r_r}{2} + I_{rb}^2 \dfrac{r_r}{2(2-s)}$

leaves for the mechanical power output.

$$p = I_{rf}^2 \frac{r_r(1-s)}{2s} - \frac{I_{rb}^2 r_r (1-s)}{2(2-s)} \qquad (17)$$

The torque in synchronous watts is then,

$$T = I_{rf}^2 \frac{r_r}{2s} - I_{rb}^2 \frac{r_r}{2(2-s)} \qquad (18)$$

which shows that the torque due to the backward field opposes that due to the forward field. The two torque curves and their resultant are plotted in Fig. 5.6. It is clear that the torque passes through zero a little prior to $s = 0$.

II. <u>Consideration with both the main and starting winding connected to supply</u>:

(a) <u>General:</u>

Wayne, J. Morrill in his classic paper on capacitor motor, used the revolving field theory to show how it is possible to explain and calculate the phenomena which appear in the operation of the motor.

He treated a capacitor motor as an unbalanced/ directly across the same line. First of all the equivalent circuit and general equations for an unbalanced two phase motor are obtained and then applied to capacitor motor.

/two phase motor in which both the stator phases are connected.

The actual derivation of the general equations is not done here in this dessertation, but they are just mentioned. (However the equivalent circuit is evolved here below).

These equations can be used as such for the starting characteristics of the capacitor-start motor. Further these can be adopted for the single phase operation as the special case of a capacitor motor in which the auxiliary phase carries no current.

(b) Equivalent circuit:

In the revolving field theory of single phase motors, the pulsating sinusoidal flux produced by the stator winding m, is resolved into two equal sinusoidal waves of flux glinding in opposite directions around the periphery of air-gap at synchronous speed. The effect upon the stator produced by each of these revolving fluxes and the induced forward and backward revolving rotor fluxes can be represented by a parallel circuit of two branches. One branch is the magnetizing reactance and the other is composed of the secondary leakage reactance in series with the secondary resistance devided by the slip as shown in fig. 3. The primary impedance is also represented by $r_{sm} + jx_{sm}$.

However there is another winding (s winding) displaced in space by 90^{o} from the m winding. The pulsating field produced by this winding also can be resolved into two

oppositely revolving fields and an equivalent circuit for this can also be drawn similarly.

The equivalent circuit as shown in figure 5.3 is valid individually for both the windings when the two windings are excited not simultaneously. For both windings it can be drawn as shown in figure 5.7.

However in an actual single phase motor with a start winding, the 4 fluxes super-impose without distortion and the equivalent circuits are the same as before except that in addition to the forward and backward voltages induced in each phase there are forward & backward voltages due to the fluxes of the other phase. Assuming that the m phase is displaced forward by 90 electrical degrees from the s phase in space, the voltage generated in the m phase by the s- forward flux must lag 90 time degrees from the voltage which the same (s forward) flux produces in the s phase; and since the ratio of turns on the m phase to those on the s phase is $= \frac{1}{k}$

$$\underline{E}_{m_{sf}} = -j \frac{\underline{E}_{fs}}{k} \tag{19}$$

Similarly proceeding

$$\underline{E}_{m_{sb}} = j \frac{\underline{E}_{bs}}{k} \tag{20}$$

$$\underline{E}_{s_{mf}} = jk \, \underline{E}_{fm} \tag{21}$$

$$\underline{E}_{s_{mb}} = -jk \, \underline{E}_{bm} \tag{22}$$

Hence under this condition the equivalent circuit can be

drawn as shown in figure 5.8

(c) Equations for currents:

From the equivalent circuit:

$$\underline{V}_m = \underline{I}_m \left[(r_{sm} + r_f + r_b) + j(x_{sm} + r_f + x_b) \right] - \underline{E}_{m\,sf} - \underline{E}_{msb}$$

$$= \underline{I}_m \left[(r_{sm} + r_f + r_b) + j(x_{sm} + x_f + x_b) \right] + j\frac{1}{k}\underline{E}_{fs} - j\frac{1}{k}\underline{E}_{bs}$$

$$= \underline{I}_m \left[(r_{sm} + r_f + r_b) + j(x_{sm} + x_f + x_b) \right]$$

$$j\frac{1}{k}\underline{I}_s (k^2 r_f + jk^2 x_f) - j\frac{1}{k}\underline{I}_s (k^2 r_b + jk^2 x_b) \qquad (23)$$

$$\therefore \quad \underline{V}_m = \underline{I}_m \left[(r_{sm} + r_f + r_b) + j(x_{sm} \;\; x_f \;\; x_b) \right]$$

$$+ \underline{I}_s\, jk \left[(r_f - r_b) + j(x_f - x_b) \right] \qquad (24)$$

Similarly $\underline{V}_s$ can be found to be,

$$\underline{V}_s = \underline{I}_s \left[r_c + k^2(r_{ss} + r_f + r_b) + jk^2(x_{ss} + x_f + x_b) \right]$$

$$+ j\underline{I}_m k \left[(r_f - r_b) + j(x_f - x_b) \right] \qquad (25)$$

Solving these two equations for $\underline{I}_m$ and $\underline{I}_s$, we have

$$\underline{I}_m = \underline{V}_m \left\{ \left[r_c + k^2(r_{ss} + r_f + r_b) \right] + j\left[x_c + k^2(x_{ss} \;\; x_f \;\; x_b) \right] \right\}$$

$$+ \frac{j\underline{V}_s k \left[(r_f - r_b) + j(x_f - x_b) \right]}{\underline{X}}$$

$$\underline{I}_s = \frac{\underline{V}_s\left[(r_{sm}+r_f+r_b)+j(x_{sm}+x_f+x_b)\right]-j\underline{V}_m k\left[(r_f-r_b)+j(x_f-x_b)\right]}{\underline{X}}$$

where

$$\underline{X} = \left\{\left[r_c+k^2(r_{ss}+r_f+r_b)\right]+j\left[x_c+k^2(x_{ss}+x_f+x_b)\right]\right\} \cdot \left\{(r_{sm}+r_f+r_b)+j(x_{sm}+x_f+x_b)\right\}-k^2\left[(r_f-r_b)+j(x_f-x_b)\right]^2 \qquad (28)$$

The values for r_f, x_f, r_b can be obtained by actually solving the concerned part of the network shown in Fig. 7

These can be shown to be

$$r_f = \frac{1}{2}\,\frac{x_m^2\,\frac{r_r}{s}}{\left(\frac{r_r}{s}\right)^2+(x_r+x_m)^2}$$

$$x_f = \frac{x_m}{2}\,\frac{\left(\frac{r_r}{s}\right)^2+x_r(x_r+x_m)}{\left(\frac{r_r}{s}\right)^2+(x_r+x_m)^2}$$

$$r_b = \frac{1}{2}\,\frac{x_m^2\left(\frac{r_r}{2-s}\right)}{\left(\frac{r_r}{2-s}\right)^2(x_r+x_m)^2}$$

$$x_b = \frac{x_m}{2}\,\frac{\left(\frac{r_r}{2-s}\right)^2\;x_r(x_r+x_m)}{\left(\frac{r_r}{2-s}\right)^2+(x_r+x_m)^2} \qquad (29)$$

(d) <u>Torque:</u>

It has been shown by Morrill that the general equation for the torque of an unbalanced two-phase motor.

$$T = I_m^2 \left[(r_f-r_b) + (r_f - r_b) \cos 2\,wt - (x_f-x_b) \sin 2\,wt\right]$$

$$I_s^2k^2 \quad (r_f-r_b) \cos 2(\phi + wt) - (x_f-x_b) \sin 2(\phi + wt)$$

$$+\ 2I_mI_sk\ (r_f+r_b) \sin \phi \tag{30}$$

Hence the starting torque of the capacitor start motor can be found from this formula since at starting the capacitor-start motor is an unbalanced two phase motor.

It can be noticed that some of the torque components are constant while others vary sinusoidally with time at twice line frequency.

Over a period of time corresponding to any number of complete cycles of the line voltage the alternating torques integrate to zero and the average torque is:

$$T_{(avg)} = (I_m^2 + k^2I_s^2)(r_f-r_b) + 2I_mI_s\,k(r_f+r_b) \sin \phi \tag{31}$$

The maximum value of alternating torque is

$$T_{(alt.max.)} = \sqrt{\left[I_m^4 + k^4I_s^4 + 2I_m^2I_s^2k^2 \cos 2\phi\right]\left[(r_f-r_b)^2 + (x_f-x_b)^2\right]} \tag{32}$$

Hence for running condition of the capacitor start motor the torque equation can be written as below (by setting $I_s = 0$ in the equations 30,31 and 32).

$$T = I_m^2 \left[(r_f-r_b) + (r_f-r_b) \cos 2\,wt - (x_f-x_b) \sin 2\,wt\right] \tag{33}$$

$$T_{(avg)} = I_m^2 (r_f - r_b) \qquad (34)$$

$$T_{(alt.\ max.)} = I_m^2 \sqrt{(r_f - r_b)^2 + (x_f - x_b)^2} \qquad (35)$$

In the equation No. 33 for the torque under running condition, one component $(r_f - r_b) \cos 2\,wt$, of the alternating torque is proportional to the average or useful torque of the motor and therefore is not controllable by design. The other component $(x_f - x_b) \sin wt$ is greatest on light loads and is subject to some control by design though an attempt at such control would probably be too costly for practical purposes.

(e) <u>Out put:</u>

The general equation for the out put of an unbalanced two-phase motor is:

$$W.O = \left[(I_m^2 + k^2 I_s^2)\ (r_f - r_b) + 2 I_m I_s k (r_f + r_b) \sin \phi\right](1 - s) - W_f \qquad (36)$$

The output of the single phase motor is therefore,

$$W.O. = I_m^2 (r_f - r_b)\ (1 - s) - W_f \qquad (37)$$

(f) <u>Input:</u>

The input can be calculated as the product of the voltage impressed on each phase by the in-phase current plus the iron loss.

III. <u>Performance calculation based on revolving field theory" Apparent Impedance" method.</u>

By revolving field theory, the apparent resistance and reactances are: as shown in equation 29. They are stated here below:

$$r_f = \frac{1}{2}\,\frac{x_m^2\,\frac{r_r}{s}}{\left(\frac{r_r}{s}\right)^2 + (x_r + x_m)^2} \cdot x_f$$

$$x_f = \frac{x_m}{2}\,\frac{\left(\frac{r_r}{s}\right)^2 + x_r(x_r + x_m)}{\left(\frac{r_r}{s}\right)^2 + (x_r + x_m)^2}$$

$$r_b = \frac{1}{2}\,\frac{x_m^2\left(\frac{r_r}{2-s}\right)}{\left(\frac{r_r}{2-s}\right)^2 + (x_r + x_m)^2} \cdot x_b$$

$$x_b = \frac{\frac{x_m}{2}\left(\frac{r_r}{2-s}\right)^2 + x_r(x_r + x_m)}{\left(\frac{r_r}{2-s}\right)^2 + (x_r + x_m)^2}$$

Each of the above 4 equations are functions of independant variables if $\frac{r_r}{s}$ and $\frac{r_r}{2-s}$ are considered as a single variable. Functions involving three independent variables of the preceding equations be expressed as per unit of one of the independent variables, it is possible to reduce the number of independent variables by one. This can be done if both sides of the above 4 equations be divided by $\left(\frac{x_m}{2}\right)$ and both the numerator and denumerator of each of the r.h.s. member of the 4 equations are divided by

$\left(\frac{x_m}{2}\right)^2$. The resulting equations are then

$$R_{rf} = \frac{r_f}{x_m/2} = \frac{\dfrac{r_r}{s\,x_m}}{\left(\dfrac{r_r}{sx_m}\right)^2 + \left(\dfrac{x_r}{x_m} + 1\right)^2}$$

= per unit forward field apparent resistance. (38)

$$x_{rf} = \frac{x_f}{x_m/2} = \frac{\left(\dfrac{r_r}{sx_m}\right)^2 + \dfrac{x_r}{x_m}\left(\dfrac{x_r}{x_m} + 1\right)}{\left(\dfrac{r_r}{sx_m}\right)^2 + \left(\dfrac{x_r}{x_m} + 1\right)^2}$$

= per unit forward field apparent reactance. (39)

$$R_{rb} = \frac{r_b}{x_m/2} = \frac{\dfrac{r_r}{x_m\,(2-s)}}{\left[\dfrac{r_r}{x_m(2-s)}\right]^2 + \left[\dfrac{x_r}{x_m} + 1\right]^2}$$

= per unit backward apparent resistance (40)

$$X_{rb} = \frac{x_b}{x_m/2} = \frac{\dfrac{r_r}{x_m(2-s)}^2 + \dfrac{x_r}{x_m}\left(\dfrac{x_r}{x_m} + 1\right)}{\left[\dfrac{r_r}{x_m(2-s)}\right]^2 + \left[\dfrac{x_r}{x_m} + 1\right]^2}$$

= per unit backward field apparent reactance. (41)

In the above equations, we will define the independent variables as below:

$$\frac{r_r}{s\,x_m} = \text{per unit secondary resistance at slip } s \qquad (42)$$

$$\frac{r_r}{x_m(2-s)} = \text{per unit secondary resistance at slip } (2-s) \qquad (43)$$

$$\frac{x_r}{x_m} = \text{per unit secondary reactance.} \qquad (44)$$

On the basis of these definitions of independent variables there are only two independent variables in the r.h.s. members of equations for per unit apparent resistance and reactance. Since the only difference between the expressions for per unit apparent forward field resistance and reactance and the similar expressions for the backward field is the use of $\frac{r_r}{s}$ in place of $\frac{r_r}{2-s}$, the curves for the forward and backward field apparent impedances will be identical and only two families of curves need be drawn.

In the figures (No 13·12(32)) there are shown curves of per unit apparent resistance and per unit apparent reactance plotted for definite values of per unit secondary reactance and as a function of the reciprocal of the per unit secondary resistance for slips of s or 2-s (The reciprocal of the per unit secondary resistance has been chosen as the independent variable because the resulting curves spread out in such a fashion as to make them more easily and more accurately read).

Hence the procedure to find the apparent impedances, is as follows:

First find the per unit secondary reactance, $\frac{x_r}{x_m}$, which

fixes the curve to be referred. In case the value of $\frac{x_r}{x_m}$ is not exactly the same as that of a curve, it is, of course necessary to interpolate between two adjacent curves. When the proper curve has been selected, the corresponding value of per unit apparent resistance or reactance may be read as the ordinate corresponding to the proper value of $\frac{sx_m}{r_r}$ or $\frac{x_m\ (2-s)}{r_r}$. The actual apparent resistance or reactance is obtained as the product of the per unit apparent resistance or reactance by $\frac{x_m}{2}$ the magnetising reactance.

Performance of single phase motion:

It is desirable to consider the core loss as if it were the result of a high resistance in parallel with the entire motor and to neglect the effect of the flow of the core-loss current through the primary impedance of the motor. When this approach is employed, the core loss may be introduced as an input which is added to the input determined from the idealized equivalent circuit. The two other losses which must be accounted for in making an exact single-phase motor calculation, are the rotational iron loss and the mechanical friction (friction & windage).

If it is presumed that the rotational iron loss and the mechanical friction are both known (say = Wf) then these can be accounted for if W_f be subtracted from the electrical output as determined from the idealized circuit.

REFERENCES: B9, J18, and J19.

CHAPTER NO. 6

CROSS FIELD THEORY

1. GENERAL

The single phase induction motor consists of a primary winding connected to a source of alternating potential and placed in inductive relation to a short-circuited secondary winding which can move relative to the primary winding.

A complete iron path, except for the air-gap necessary to allow relative motion between the two windings is provided for the flux which interlinks the two windings. This interlinking flux is produced by the primary winding because of its connection to a source of alternating potential, and it is the effect of this flux on the short-circuited secondary winding in which we are interested. This will be dealt with in some detail in this chapter.

2. STATOR REACTIONS:

The stator winding is so arranged that when it is energised from the supply line, the magnetic field which it produces alternates cyclically along the vertical y-axis. Let us assume that the turns of the winding are so disposed in the slots of stator core as to produce a sinusoidal space distribution of flux circumferentially around the air gap under the polar areas and that with reference to time the flux also varies sinusoidally of the total magnetic field, a -

small portion encircles only the stator inductors without crossing the air gap into the rotor. This leakage flux is considered to give rise to the leakage reactance voltage in the primary windings. The reminder of the magnetic field, the mutual flux ϕ_M interlinks the rotor as well as the stator inductors. This concept is shown in fig. 6.1.

The rotor inductors could be considered to be paired at any instant with those on the other side of the vertical plane, as shown in fig. 6.2

The periodic variation of the mutual flux ϕ_M of the main field induces a transformer voltage both in the stator winding and in the rotor winding. In the stator winding, the vector difference of this transformer voltage and impressed line voltage is the leakage impedance voltage and it is proportional to the primary current. (Its variation between no load and full load causes a small change in the magnitude of the mutual flux ϕ_M which is ignored and the flux ϕ_M is considered constant in this analysis).

3. ROTOR - TRANSFORMER VOLTAGE EFFECTIVE IN THE y AXIS:

In the rotor winding the transformer voltage which is induced there by the alternations of the main field ϕ_m is only one of the several component of voltages which are to be individually considered during the development of this theory. The amplitude of this induced emf. is determined by the magnitude of the mutual flux, by the frequency and by

the equivalent No. of turns in the rotor winding. In time phase, the voltage is in lagging quadrature with the flux ϕ_m. This voltage causes currents in the closed rotor circuits as shown by the paired inductors of Fig. 6.2. Further the induction of this transformer voltage is not influenced or modified by whether the rotor is stationary or in rotation, since, in rotation, the successive inductors momentarily form new pairs as they pass through the vertical plane, the trace of which is the vertical y-axis.

The transformer voltage induced in the rotor may be designated by appropriate subscripts; T for it results from transformer action; M for it is due to the main mutual field ϕ_m; and y for it causes a resultant mmf. along the y axis - E_{TMy}.

The direction of the induced voltage E_{TMy} and the direction of the dependent magnetic effect of its current through the resultant mmf. are more simply represented by considering all of the paired inductors in Fig. 6.2. to be replaced, (for the particular instant considered) by a coil of an equivalent number of turns and located in the horizontal plane, through the rotor axis so as to retain the same magnetic y axis, as shown in figure 6.3.

In the development of the following presentation, one particular instant of time is assumed, although it will be observed later that the same terminal results obtain by the use of any other chosen instant. The instant of time to be considered is shown by the vertical line on the time-phase diagram, in Fig. 6.4.

At this instant the curve representing the stator main field ϕ_M is increasing in magnitude, and the assumption is made that the upper stator pole in Fig. 3 is of north polarity. The voltage E_{TMy} is shown in the time phase diagram (Fig. 6.4) as curve 2, having been drawn in quadrature lagging phase with reference to the stator field ϕ_M which induces it. The direction of E_{TMy} is determined by Lenz's law. Its direction is such that its resulting magnetic effect along the y-axis opposes the increase of the stator field ϕ_M and this direction is so indicated in the space-phase diagram, Fig. 6.3.

4. <u>SPEED VOLTAGE GENERATED IN ROTOR BY MAIN FIELD</u>

When the rotor inductors rotate through the main field ϕ_M speed voltages are generated in one direction in all of those inductors which are, <u>at the instant</u>, above the horizontal plane and voltages in the opposite direction in inductors occupying positions below the horizontal plane. These inductors, may also be regarded as paired, each pair having its magnetic axis individually coincident with the X-axis as shown in fig. 6.5.

This speed voltage is identified as E_{SMx} S-standing for generation by speed action. This voltage E_{SMx} is in time-phase with the main field ϕ_M and is shown as curve (3) in Fig. 6.4. If the motor is operating on load, it has an appreciable slip and hence the speed voltage E_{SMx} is less in magnitude than the transformer voltage E_{TMy}

of curve (2) by the proportion that the speed is less than synchronous.

Therefore the value of $E_{SMx} = S \; E_{TMy}$ where S = actual rpm./synchronous rpm.

The transformer voltage, is of course, independent of the speed.

The currents in the paired conductors which result from the speed voltages, produce their mmf. and hence their resultant field ϕ_Q, along the x-axis. By reason of this quadrature axis field, the name "Cross-axis" theory is derived.

Because these paired inductors possesses the traits of an iron core inductor, with no other winding to act upon, this exciting current I_x of the rotor, that results from the speed voltage, lags ES_{MX} in time phase by nearly 90 electrical degrees. In the interest of simplicity, the quadrature field ϕ_Q is considered as being proportional to and in time-phase with the rotor exciting current I_x rather than being in phase with the magnetizing component of the exciting current a current which lags I_x by a small angle and which lags E_{SMx} by 90 electrical degrees.

The quadrature field ϕ_Q is so drawn in time phase with I_x in the Fig. 6.6, - shown as curve (5).

As shown previously, at a particular instant the quadrature flux ϕ_Q may be regarded as being caused by the

mmf. of field current I_x in an equivalent hypothetical coil which replaces in effect the mmfs. of all the paired inductors on the rotor. This equivalent hypothetical coil is shown in the fig. 6.7, and for the instant 1 when the upper main pole is of north polarity, and for a counter-clockwise direction of rotation, the direction of the generated speed voltage E_{SMx} in this coil is also shown.

The polarity of the quadrature field at instant 1 can be obtained by referring to figure 6.4. Because of the large phase angles of the rotor cross field current, I_x lags behind the speed voltage E_{SMx} which causes it, its polarity at instant 1 is directed oppositely to E_{SMx}. Hence in figure 6.7, the exciting current I_x is shown inward for the inductors at the top of the equivalent coil, as against outward for the voltage, E_{SMx}. The vector diagram for these are shown in Fig. No. 6.7(a).

5. <u>Transformer voltage induced in rotor by quadrature field:</u>

Fig. 6.6 shows the curve of quadrature field (curve 5) to be decreasing toward zero at instant 1, and, because of this, transformer voltages are being induced in the same group of inductors in which speed voltages from the main field ϕ_m are being generated. By the application of Lenz's law, the induction is found to be in such a direction as to sustain the decreasing quadrature field, and thus these induced voltages are seen to be oppositely directed to those generated by speed action from the main field. The voltages are inward

for the inductors above the x-axis and outward below. This voltage E_{TQx} is shown in figure 6.7.

The transformer voltage E_{TQx} which is induced by the quadrature field ϕ_Q lags the flux by 90 electrical degrees. The magnitude of E_{TQx} is almost equal to that of the speed voltage, E_{SMx} being less only by the vector difference of the rotor leakage impedance voltage which results from the small rotor current I_x. On the time-phase diagram this transformer voltage E_{TQx} is drawn as curve (6) in lagging quadrature with ϕ_Q and in magnitude a little less than that of E_{SMx}. Because these two voltage curves are quite in phase opposition, the value of E_{TQx} at instant 1 is lightly greater than the value of E_{SMx}, a fact which accounts for the rotor exciting current I_x, being momentarily in the direction opposite to E_{SMx}, as previously described and shown in Fig.7. Hence the vector diagram can therefore be drawn as in Fig. No. 6.7(b).

6. Speed voltages are generated in those inductors which are under the influence of the quadrature poles and which inductors are considered as being paired coaxially about the y-axis. These are the same paired inductors which are shown in fig. 6.2, as having induced in them the transformer voltage E_{TMy} from the main field ϕ_M. This voltage, E_{SQy} is found to have a direction opposite to that of the transformer voltage E_{TMy}.

The time phase of the speed voltage E_{SQy} with reference

to the quadrature field, whether in phase with it or in phase opposition it, has to be determined. This can be found from the fact that the direction of the speed voltage E_{SQy} is opposite to that of the transformer voltage E_{TMy}.

The direction of E_{TMy}, Fig. 6.4, at instant 1, is the same as the quadrature field, Fig. 6.6. Hence the speed voltage E_{SQy} is drawn in phase opposition to the quadrature field ϕ_Q, which causes it.

Since the transformer voltage E_{TMy} and the speed voltage E_{SQy} appear as oppositely directed voltage components in the same group of rotor inductors, Fig. 6.8, their vector difference is then the unbalanced leakage impedance voltage drop which causes the load current I_y in those inductors, coaxially grouped about the y-axis.

The transformer voltage E_{TMy} being induced by the assumed constant main field ϕ_M, is independent of speed, and hence, it is the largest voltage component in the rotor. The speed voltage E_{SQy} is the smallest since it is dependent upon the magnitude of the quadrature field which is less than that of the main field by the approximate value of the rotor speed compared with that of the synchronous speed, and also it is dependent upon the speed of cutting the quadrature field which is likewise less than the synchronous speed. In other words, the speed voltage E_{SQy} is about proportional to the square of the rotor speed.

Thus the curve of the speed voltage appears as E_{SQy} (7) drawn in time phase opposition to the field ϕ_Q (5) and to a magnitude less than that of E_{TQx} (by approximate ratio of $\frac{\text{rotor speed}}{\text{syn. speed}}$) and less than that of E_{TMy} (by the square of the above ratio of speeds).

So we can represent these quantities in a vector diagram as shown in fig. No. 6.8(a).

7. <u>The load current</u> I_y <u>of the rotor:</u>

The load current I_y in those equivalent rotor inductors as shown in Fig. 6.8, is caused by the transformer voltage E_{TMy} and the magnitude of the current is so controlled by the countervoltage. E_{SQy} as to accommodate itself to variations in the load. As the load on the motor increases and with it the speed voltage E_{SQy} decreases. Thus with decreasing speed the vector voltage difference between the constant voltage E_{TMy} and the variable speed voltage E_{SQy} increases and in proportion to it, the load current I_y increases and simultaneously improves its p-f with respect to E_{TMy}. This load current I_y is shown in suitable magnitude and phase in fig. 6.6, as curve (8). It lags E_{TMy} by a small angle for heavy load and by a large angle for light load.

Now we can draw the phaser diagram for the stator as shown in Fig. 6.9, where I_m is the total primary current and I_ϕ is the current in the magnetising impedance z_m.

From this figure, we have for E_{TMy}

$$E_{TMy} = I_\phi \; z_m = -(I_m - I_y)z_m \qquad (1)$$

Now applying Kirchoff's law to the stator

$$-E_{TMy} + I_m z_s = V_s$$

$$\therefore \quad V_s = I_m z_s + (I_m - I_y) z_m \qquad (2)$$

8. Rotor voltages due to leakage fluxes:

Now we have to consider the two rotor leakage fluxes ϕ_{LM} and ϕ_{LQ}. The rotor current I_y will provide a leakage flux ϕ_{LM} while the rotor current I_x will provide a leakage flux ϕ_{LQ}.

The rotor conductors in the y-axis will cut ϕ_{LM} while the conductors in the x-axis will cut the flux ϕ_{LQ}. Therefore, two additional speed voltages are generated. At synchronous speed these speed voltages would be numerically equal to leakage reactance drops. For any speed less than synchronous the voltages will be less by the ratio S.

The voltage effective in the y-axis is that due to the cutting of the flux ϕ_{LQ}.

$$E_{SLQy} = S \; I_x \; x_r \qquad (3)$$

The voltage effective in the x-axis is that due to the cutting of ϕ_{LM}.

$$E_{SLMx} = S\, I_y\, x_r \tag{4}$$

9. Rotor voltages:

All the rotor voltages considered till now are enumerated here below:

(1) E_{TMy} (2) E_{SMx} (3) E_{TQx} (4) E_{SQy} (5) E_{SLMx}

(6) E_{SLQy} (5)

Of these total voltages effective in the x-axis

$$= E_{rx} = E_{TQx} + E_{SMx} + E_{SLMx} \tag{6}$$

Total voltage effective in the y-axis

$$= E_{ry} = E_{TMy} + E_{SQy} + E_{SLQy} \tag{7}$$

These resultant induced voltages in each axis (E_{rx} and E_{rf}) must be equal to the impedance drops of that particular circuit.

$$\therefore\ E_{rx} = E_{TQx} + E_{SMx} + E_{SLMx} = I_x Z_r \tag{8}$$

$$E_{ry} = E_{TMy} + E_{SQy} + E_{SLQy} = I_y z_r \tag{9}$$

The speed voltages depend on the maximum values of the respective fluxes and the actual speed of the rotor, while the transformer voltages depend only on the maximum value of the proper flux. Summarising we may state that voltages E_{TMy} and E_{SMx} are produced by flux ϕ_m and

E_{TQx} and E_{SQy} are produced by flux ϕ_Q. Further, E_{SMx} is in time phase with ϕ_M whereas E_{TMy} is in phase quadrature lag to ϕ_M.

Also E_{SQy} is in phase opposition to flux ϕ_Q while E_{TQx} is 90 time degrees behind ϕ_Q.

Therefore, $E_{SMx} = j\,S\,E_{TMy}$ (10)

$E_{SQy} = -j\,S\,E_{TQx}$ (11)

where S = ratio of actual rotor speed to the synchronous speed.

We also know that

$E_{TQx} = I_x z_m$ (12)

$\therefore\ E_{SQy} = -j\,S E_{TQx} = j\,S\,I_x z_m$ (13)

Substituting the values for E_{TQx}, E_{SMx} and E_{SLMx} from equations 12, 10 and 4 in the equation 8 we get.

$\therefore\ I_x z_m + j\,S\,E_{TMy} + S\,I_y\,x_r = I_x z_r$ (14)

Substituting the values for E_{SQy} and E_{SLQy} from equations 11, & 3 in equation 9, we get,

$E_{TMy}\quad j\,SI_x z_m - SI_x x_r = I_y z_r$ (15)

10. <u>Currents:</u>

To refer these quantities to stator we have to put $-I_y$ in place of I_y

Also substituting for e_{TMy} the expression in equation (1) we have

$$- I_x z_m - j\, S(I_m - I_y) z_m - S\, I_y x_r = I_x z_r \qquad (16)$$

$$- (I_m - I_y) z_m + j\, S I_x z_m - S I_x\, x_r = - I_y z_r \qquad (17)$$

We already know from the equation No. 2 that

$$I_m z_s + (I_m - I_y) z_m = V_s \qquad (2)$$

These 3 equations Nos. 2, 16 and 17 form simultaneous equations which can be solved for I_m, I_x & I_y.

However an approximation will now be made in order to simplify calculations. The core losses will be neglected initially so that $z_m = j x_m$. The core loss can be accounted for by any one of the methods suggested in the next section. If this value is substituted the 3 equations shown in 2, 16 and 17 become

$$S x_m (I_m - I_y) - j\, x_m - S I_y\, x_r = I_x\, z_r \qquad (18)$$

$$j x_m (I_m - I_y) + S\, I_x\, (x_m + x_r) = I_y\, z_r \qquad (19)$$

$$I_m z_s + j\, x_m (I_m - I_y) = V_s \qquad (20)$$

and the vector diagrams for stator, rotor quadrature axis and the rotor main axis change to those shown in figure 10. If the above 3 equations are solved for I_m, I_x and I_y, we get,

$$I_m = - V_s \frac{-r_r^2 + (1-S^2)\, x_o^2 - j\, 2 r_r x_o}{U + jW} \qquad (21)$$

$$I_y = V_s x_m \frac{(1 - S^2)\, x_o - j\, r_r}{U + jW} \qquad (22)$$

$$I_x = V_s \frac{S\, x_m\, r_r}{U + jW} \qquad (23)$$

Wherein

$$U = r_s r_r^2 + 2r_r x_s x_o + r_r x_m (x_o + x_r) + (1-S^2) r_r x_o^2 \qquad (24)$$

$$W = - r_r^2 x_s - 2r_r r_s x_o - r_r^2 x_m + (1-S^2)(x_s x_o^2 + x_r x_m x_o) \qquad (25)$$

where $x_o = x_m + x_r$ (26)

11. <u>Treatment of core loss:</u>

(a) The core loss can be treated as a loss in a resistance connected in series with the magnetising impedance x_m in each axis. The correct calculation of this resistance results in a copper loss equivalent to half of the core loss, the basic idea being that approximately half of the core loss occurs in each axis.

(b) Another method which does not interfere with the equations for I_m, I_y and I_x already arrived at, but which is only approximate, is based on the idea of subtracting half of the core loss from the rotor output and adding the other half to the stator input.

(c) A third method is the simple addition of the entire core loss to the stator input, adding an appropriate in-place component of current.

12. Torque:

The torque of the single phase induction motor can be obtained by subtracting the rotor losses from the rotor input and dividing the resulting output by S. This gives the torque in synchronous watts.

The more direct method takes into account the fact that torque results from the effect of the current in the y-axis upon the flux ϕ_Q and from the effect of the current in the x-axis upon the flux ϕ_M.

ϕ_Q is proportional to $I_x\,(x_m + x_r) = I_x\,x_o$

The numerical value of $I_x x_o$ is

$$\frac{V_s\,x_m\,r_r\,x_o\,S}{\sqrt{U^2 + W^2}} \quad \text{volts.}$$

The inphase component of current I_y is,

$$V x_m \frac{x_o\,(1 - S^2)}{\sqrt{U^2 + W^2}} \quad \text{amps.} \qquad (28)$$

Therefore the torque due to $\phi_Q . I_x$

$$T_u = \frac{V_s^2\,x_m^2 x_o^2\,r_r\,S\,(1 - S^2)}{U^2 + W^2} \quad \text{in synchronous watts} \qquad (29)$$

The x-axis current and the M-axis flux react to produce a retarding torques, which is a measure of the core losses (which would include the iron losses of that axis had they been considered in the equations).

With the leakage flux considered the M-axis flux is propor-

proportional to

$x_m (I_m - I_y) - I_y x_r$

$= \quad x_m I_m - I_y x_o$

but $\quad x_m I_m - I_y x_o \quad = x_m \left[-V_s \dfrac{-r_r^2 + (1 - S^2)x_o^2 - j2r_r x_o}{U + jW} \right]$

$$- x_o \left[-V_s x_m \frac{(1-S^2)x_o - jr_r}{U + jW} \right] \qquad (30)$$

Combining and using only the real terms (The current I_x with which ϕ_M is associated contains no j term since it is in the axis of reference for this calculation. Since the projection of ϕ_M on i_x is all that is significant in producing torque, the j terms are omitted), the final expression is

$$\frac{V_s x_m r_r^2}{\sqrt{U^2 + W^2}} \qquad (31)$$

This reacts with $I_x \quad = \dfrac{V_s S \; x_m r_r}{\sqrt{U^2 + W^2}}$ (32)

to produce the retarding torque in synchronous watts.

$$T_{ret} = \frac{V_s^2 \; x_m^2 \; r_r^2 S}{U^2 + W^2} = \text{quadrature axis losses} \qquad (33)$$

$\therefore$ The net torque developed is then their difference.

$$T = \frac{V_s^2 \; x_m^2 \; r_r S \quad (1-S^2)x_o^2 - r_r^2}{U^2 + W^2} \text{ Synchronous watts} \qquad (34)$$

<u>REFERENCES</u> B7, B10, J20 and J21.

CHAPTER NO. 7

EQUIVALENT CIRCUIT AND CIRCLE DIAGRAM BASED ON DOUBLE REVOLVING FIELD AND CROSS FIELD THEORY

I. Introduction:

In 1930, Gabriel Kron presented a generalised theory of electric machinery based upon energy content and energy utilisation. In his paper he considers an electric machine as a device which receives energy and through a process of energy storage and energy transmission finally delivers a part of the total input energy into a load. A similar development is presented by Tarboux in his paper on "generalised circle diagram for a 4 terminal network" however from the circuit point of view.

In general a four-terminal network is defined as any electric circuit composed of resistances, inductances and capacitances so connected that there is only one point of energy input and only one point for energy output. For normal analytical treatment it is customary to assume that this network is linear and bilateral.

From energy point of view the single phase motor as well as the 3 phase induction motor, corresponds to a four-terminal net work- the input being electric and the output delivered in mechanical form. The basic problem is therefore, to

represent the mechanically obsorbed power by an equivalent electrical impedance-element and to incorporate in the net work a suitable circuit which represents the electrical and magnetic features of the motor.

The figure No. 7.1, indicates a generalised circuit which is defined by the use of the so-called generalized constants A,B C and D in the form of two generalized equations.

$$\underline{V}_1 = \underline{A}\,\underline{V}_r + \underline{B}\,\underline{I}_r$$

$$\underline{I}_1 = \underline{C}\,\underline{V}_r + \underline{D}\,\underline{I}_r \qquad (1)$$

The four terminal net work may be a simple series net work, a simple shunt impedance, a T circuit as used for transformers and 3 phase induction motors (per phase) or any other combination of resistances, inductances and capacitances.

2. Development of generalized circle diagram equations.

$$V_1 = A\,V_r + B\,I_r \qquad (2)$$

$$I_1 = C\,V_r + D\,I_r \qquad (3)$$

But $V_r = I_r Z$

$$\therefore \quad V_1 = I_r AZ + I_r B$$

$$I_1 = I_r CZ + I_r D$$

$$\therefore \quad \frac{I_1}{V_1} = \frac{CZ + D}{AZ + B}$$

$$\therefore \quad I_1 = V_1 \left[\frac{CZ + D}{AZ + B}\right] \tag{4}$$

It can be shown for linear bilateral net works that

$$AD - BC = 1 \tag{5}$$

$$\therefore AD = BC + 1$$

$$I_1 = V_1 \frac{ACZ + AD}{A^2Z + AB} = V_1 \frac{ACZ + BC + 1}{A^2Z + AB}$$

$$I_1 = V_1 \frac{C(AZ + B) + 1}{A(AZ + B)} \tag{6}$$

$$\therefore \quad I_1 = \frac{V_1 C}{A} + \frac{V_1}{A^2 Z \quad AB} = \frac{V_1 C}{A} + \frac{\frac{V_1}{A^2}}{\frac{B}{A} \quad Z} \tag{7}$$

This equation for I_1 is the generalized circle diagram equation.

3. <u>By revolving field Theory</u>:

It has been shown in the "Revolving field theory" Chapter that the single phase motor with one winding can be represented by the equivalent circuit shown in fig. 7.2 As it stands this equivalent circuit is a six-terminal network. Hence this has to be reduced to a four terminal network in order to apply the generalized circuit equations in terms of generalized circuit constants A,B,C and D.

$$Z_1 = \frac{r_r}{2s} + j\frac{x_r}{2} \qquad (8)$$

$$Z_2 = \frac{r_r}{2(2-s)} + j\frac{x_r}{2} \qquad (9)$$

The equivalent circuit may be reduced to simpler form.

$$\frac{1}{Z_{AB}} = \frac{1}{z_1} + \frac{1}{\frac{z_m}{2}} -$$

$$\therefore \quad Z_{AB} = \frac{z_m z_1}{z_m + 2z_1} \qquad (10)$$

Similarly,

$$Z_{BC} = \frac{z_m z_2}{z_m + 2z_2} \qquad (11)$$

$$\therefore \quad Z_{AC} = \frac{z_m z_1}{z_m + 2z_1} + \frac{z_m z_2}{z_m + 2z_2}$$

$$= \frac{z_m^2 z_1 + 4z_m z_1 z_2 + z_m^2 z_2}{z_m^2 + 2z_m(z_1 + z_2) + 4z_1 z_2} \qquad (12)$$

In terms of admittance,

$$Y_{AC} = \frac{z_m^2 + 2z_m(z_1 + z_2) + 4z_1 z_2}{z_m^2 z_1 + 4z_m z_1 z_2 + z_m^2 z_2}$$

$$= \frac{z_m^2 + 2z_m(z_1 + z_2) + 4z_1 z_2}{z_m \left[z_m(z_1 + z_2) + 4z_1 z_2\right]}$$

$$= \frac{z_m^2 + z_m(z_1 + z_2)}{z_m\left[z_m(z_1 + z_2) + 4z_1z_2\right]} \quad \frac{z_m(z_1 + z_2) + 4z_1z_2}{z_m\left[z_m(z_1 + z_2) + 4z_1z_2\right]} \tag{13}$$

$$= \frac{1}{z_e} + \frac{1}{z_m}$$

where Z_e $= \dfrac{z_m(z_1 + z_2) + 4z_1z_2}{z_m + z_1 + z_2}$ (14)

Now from the initial definitions of the several impedances,

$$z_1 + z_2 = \frac{r_r}{2s} + j\frac{x_r}{2} + \frac{r_r}{2(2-s)} + j\frac{x_r}{2}$$

$$= jx_r + \frac{(2-s)r_r + sr_r}{2s\ (2-s)}$$

$$= jx_r + \frac{r_r}{s(2-s)} = \frac{r_r + jx_r\ s(2-s)}{s(2-s)} \tag{15}$$

$$z_1z_2 = \left(\frac{r_r}{2s} + j\frac{x_r}{2}\right)\left(\frac{r_r}{2(2-s)} + j\frac{x_r}{2}\right)$$

$$= \frac{r_r^2 - x_r^2 s\ (2-s) + 2j\ x_r\ r_r}{4s\ (2-s)} \tag{16}$$

Using these results, we obtain for z_e

$$z_e = \frac{r_r^2 - x_r^2 s(2-s) + 2jx_r r_r + z_m\left[r_r + jx_r s(2-s)\right]}{z_m s(2-s) + r_r + jx_r s(2-s)}$$

$$= \frac{jx_r \left[z_m\, s(2-s) + r_r + jx_r s(2-s)\right] + r_r^2 + z_m r_r + jx_r r_r}{z_m s(2-s) + r_r + jx_r s\,(2-s)}$$

$$= \frac{r_r^2 + z_m r_r + jx_r r_r}{z_m s(2-s) + r_r + jx_r s(2-s)} + jx_r \tag{17}$$

$$= jx_r + z_g \quad \text{where} \quad z_g = \frac{r_r^2 + z_m r_r + jx_r r_r}{z_m s(2-s) + r_r + jx_r s(2-s)} \tag{18}$$

Hence the equivalent circuit can be redrawn as in Fig.7.3

Further $Y_g = \dfrac{1}{z_g} = \dfrac{(jx_r + z_m)s(2-s) + r_r}{r_r\left[r_r + jx_r + z_m\right]}$

By adding $s(2-s)\ r_r$ and subtracting in the numerator of the last fraction,

$$Y_g = \frac{(r_r + jx_r + z_m)\ s(2-s) + r_r - s(2-s)\ r_r}{r_r\ (r_r + jx_r + z_m)}$$

$$= \frac{s(2-s)}{r_r} + \frac{(1-s)^2}{(r_r + jx_r + z_m)}$$

$$= Y_b + Y_a$$

$$z_a = \frac{Y_r + jx_r + z_m}{(1-s)^2} = \frac{z_r + z_m}{(1-s)^2} \tag{19}$$

$$z_b = \frac{r_r}{s(2-s)} \tag{20}$$

Hence the equivalent circuit can be further modified as in Fig. 7.4. Still another form of equivalent circuit may be obtained through a further manipulation of equation for Y_g.

$$Y_g = \frac{s(2-s)}{r_r} + \frac{1-2s+s^2}{r_r + jx_r + z_m}$$

$$= \frac{s(2-s)}{r_r} - \frac{s(2-s)}{r_r + jx_r + z_m} + \frac{1}{r_r + jx_r + z_m}$$

$$= \frac{s(2-s)\ (jx_r + z_m)}{r_r(r_r + jx_r + z_m)} + \frac{1}{r_r + jx_r + z_m}$$

$$= Y_c + Y_d$$

where $$z_c = \frac{1}{Y_c} = r_r + jx_r + z_m = z_r + z_m \qquad (21)$$

and $$z_R = \frac{1}{Y_d} = \frac{r_r(r_r + jx_r + z_m)}{s(2-s)\ (j\,x_r + z_m)}$$

$$= \frac{r_r\ (z_r + z_m)}{s(2-s)\ (z_m + jx_r)} \qquad (22)$$

Hence the equivalent circuit is as shown in Fig. 7.5

This circuit is seen to be a four-terminal network, of which the input terminals are at a and b and output terminals are at c and d.

4. By Cross Field theory:

From the analysis by cross-field theory, the basic equations for rotor voltages in the two axes are:

$$-I_x z_m + jS\, E_{TMy} + S.\, I_y x_r = I_x z_r$$

$$E_{TMy} + j\, SI_x z_m - SI_x x_r = I_y z_r$$

These equations can be solved for I_y and I_x in terms of the transformer voltage E_{TMy}. Since all the rotor data has been interpreted on the basis of the primary or stator, it follows that

$$E_{TMy} = E_1 = \text{primary back emf.}$$

The applied voltage V_s differs from the primary back emf. by the amout of the primary impedance drop.

Hence the first element of the equivalent circuit must be an impedance z_s through which flows a current I_m. The main exciting current I_ϕ must flow through the exciting impedance z_m. And the current I_y which is the primary equivalent of rotor current I_y' must flow through the rest of the circuit which has been denoted as a single impedance z_e. The voltage across z_m is equal to the primary back emf. $E_1 = E_{TMy}$. Hence the complete equivalent circuit can therefore be specified by a detailed statement of z_e which must be equal to $z_e = \dfrac{E_{TMy}}{I_y}$.

It may be asked at this stage, whether the current I_x will not come in the equivalent circuit. The answer is that it will not come, as the equivalent circuit is drawn as viewed from the primary and only the main flux $\emptyset_M$ (which is affected by only I_y and not by I_x) involves the maximum possible magnetic coupling with the primary winding axis. The quadrature flux $\emptyset_Q$ (which is produced by I_x) has <u>Zero-coupling</u>, with the stator winding since its axis is in quadrature with the stator winding axis.

$$-I_x z_m + jSE_{TMy} + SI_y x_r = I_x z_r$$

$$E_{TMy} + jSI_x z_m - S I_x x_r = I_y z_r$$

Solving for I_y

$$I_x (z_r + z_m) = SE_{TMy} + SI_y x_r$$

$$\therefore I_x = \frac{1}{(z_r + z_m)} (j SE_{TMy} + SI_y x_r)$$

$$E_{TMy} + I_x S(j z_m - x_r) = I_y z_r$$

$$\therefore I_y = \frac{E_{TMy}}{z_r} + \frac{S}{z_r}(j z_m - x_r)\left[\frac{j SE_{TMy}}{z_r + z_m}\right]$$

$$+ \frac{S}{z_r} \frac{(j z_m - x_r) S x_r I_y}{(z_r + z_m)}$$

$$\therefore I_y \left[1 - \frac{x_r S^2 (j z_m - x_r)}{z_r(z_r + z_m)}\right] = \frac{E_{TMy}}{z_r} + \frac{jS^2 E_{TMy}(jz_m - x_r)}{z_r(z_r + z_m)}$$

$$I_y \quad \frac{z_r(z_r + z_m) - x_r S^2 \; (j\, z_m - x_r)}{z_r(z_r + z_m)}$$

$$= \frac{E_{TMy}\;(z_r + z_m) + j\; S^2\; E_{TMy}\;(jz_m - x_r)}{z_r(z_r + z_m)}$$

$$\therefore I_y = \frac{E_{TMy}\;\left[(z_r + z_m) + jS^2(jz_m - x_r)\right]}{\left[z_r\;(z_r + z_m) - S^2\; x_r(jz_m - x_r)\right]} \qquad (22)$$

$$\therefore \frac{1}{z_e} = \frac{(z_r + z_m) + jS^2\;(j\; z_m - x_r)}{z_r\;(z_r + z_m) - S^2 x_r\;(jz_m - x_r)}$$

$$\text{slip} \quad s = \frac{N_s - N}{N_s} = 1 - \frac{N}{N_s} = 1 - S$$

$$\therefore S = (1 - s)$$

$$\frac{1}{z_e} = \frac{(z_r + z_m) + j(1-s)^2(jz_m - x_r)}{z_r\;(z_r + z_m) - (1-s)^2 x_r\;(jz_m - x_r)}$$

$$\therefore z_e = \frac{z_r(z_r + z_m) - (1-s)^2\; x_r\;(jz_m - x_r)}{(z_r + z_m) + j(1-s)^2\;(jz_m - x_r)} \qquad (23)$$

$$= \frac{z_r^2 + z_r z_m - (1-s)^2(j\; x_r z_m) + x_r^2(1-s)^2}{(z_r + z_m) - z_m(1-s)^2 - jx_r(1-s)^2}$$

$$= \frac{z_m\left[z_r - (1-s)^2 9\;(jx_r)\right] + z_r^2 + (1-s)^2\; x_r^2}{z_m\;\left[1 - (1-s)^2\right] + z_r - jx_r\;(1-s)^2}$$

$$= \frac{z_m\left[r_r + jx_r - (1-2s+s^2)\,jx_r\right] + (r_r + jx_r)^2\;(1-2s+s^2)x_r^2}{z_m\;\left[1-(1-2s+s^2)\right] + (r_r + jx_r - jx_r(1-2s+s^2)}$$

$$\therefore z_e = \frac{z_m\left[r_r + jx_r(2-s)s\right] + (r_r^2 + 2jr_rx_r - x_r^2(2-s)s}{z_m(2-s)s + r_r + jx_r(2-s)s} \qquad (24)$$

The same equation for the equivalent impedance was obtained by using the double-revolving field theory (please refer to equation No. 17) thus proving the equivalence of the two theories:

5. <u>Circle diagram:</u>

It has been proved that a correct four terminal net work as shown in fig. 7.5, can represent a single-phase motor as developed by either the cross-field theory or revolving field theory.

The receiver terminals c and d of this circuit are loaded with an impedance, Z_R.

$$z_R = \frac{V_r(z_r + z_m)}{s(2-s)(z_m + jx_r)}$$

The generalized circuit equations of all linear, bilateral four terminal net works have been given in equation (2) and (3). Applying these to the net work shown in fig.7.5

$$V_s = A\,V_r + B\,I_R \qquad (25)$$

$$I_m = CV_r + DI_R \qquad (26)$$

Also $$I_m = I_\phi + I_y \qquad (27)$$

$$I_y = I_c + I_R \qquad (28)$$

$$\therefore V_s = I_m Z_s + jI_y x_r + V_r \tag{29}$$

Further $$I_c = \frac{V_r}{z_r + z_m} \tag{30}$$

$$I_\phi = \frac{V_r + jI_y x_r}{x\ z_m} = \frac{V_r + j\left[I_R + \frac{V_r}{z_r + z_m}\right] x_r}{z_m}$$

$$= \frac{V_r\ (z_r + z_m + jx_r) + jI_R x_r\ (z_r + z_m)}{z_m^2 + z_m z_r} \tag{31}$$

$$I_m = I_\phi + I_y$$

$$\therefore I_m = \frac{V_r(z_r + z_m + jx_r) + jI_R x_r (z_r + z_m)}{z_r z_m + z_m^2} + I_R + \frac{V_r}{z_r + z_m} \tag{32}$$

$$\therefore I_m = \left[\frac{z_r + 2z_m + jx_r}{z_m(z_r + z_m)}\right] V_r + \left[\frac{z_m + jx_r}{z_m}\right] I_R \tag{33}$$

Moreover $V_s = I_m z_s + jx_r I_y + V_r$

$$= \left[\frac{z_r + 2z_m + jx_r}{z_m(z_r + z_m)}\right] V_r z_s + \left[\frac{z_m + jx_r}{2_m}\right] I_R z_s$$

$$+ jx_r\left[+ I_R + \frac{V_r}{z_r + z_m}\right] + V_r$$

$$\therefore V_s = \left[\frac{z_s(z_r + 2z_m + jx_r) + z_m(z_m + z_r + jx_r)}{z_m\ (z_r + z_m)}\right] V_r$$

$$+ \left[\frac{z_s(z_m + jx_r) + jz_m x_r}{z_m}\right] I_r \tag{34}$$

Comparing equation 25 and 26 with 33 and 34 we have the A,B,C,D constants for this net work, as shown below:

$$A = \frac{z_s(z_r + 2z_m + jx_r) + z_m(z_m + z_r + jx_r)}{z_m(z_r + z_m)}$$

$$B = \frac{z_s(z_m + jx_r) + jz_m x_r}{z_m}$$

$$C = \frac{z_r + 2z_m + jx_r}{z_m\ (z_r + z_m)}$$

$$D = \frac{z_m + jx_r}{z_m} \qquad (35)$$

It has been shown that in a four-terminal net work with the generalised constants A, B, C and D, the locus of the input current vector can be represented as a circle and that this input current can be represented by the equation (applying equation 7 to the net work under consideration).

$$I_m = \frac{C}{A} V_s + \frac{\frac{V_s}{A^2}}{\frac{B}{A} + z_R} \qquad (36)$$

It can be shown that, $\frac{A}{C}$ = primary looking in impedance with the load across C and d open circuited $= z_{io}$ (37)

Also it can be shown that, $\frac{B}{A}$ = looking back

impedance as viewed from the receiver terminals (with the load removed) when the sending end terminals are short-circuited $= z_{bs}$ (38)

These two relations can also be checked by substituting the values for A, B & C and reducing the equivalent circuit.

$$\therefore I_m = \frac{V_s}{z_{io}} + \frac{\frac{V_s}{A^2}}{z_{bs} + z_R}$$

$$= \frac{V_s}{z_{io}} + \frac{V_s/A^2}{z_{bs} + \frac{V_r(z_r + z_m)}{s(2-s)(z_m + jx_r)}}$$

$$\therefore I_m = \frac{V_s}{z_{io}} + \frac{\frac{V_s}{A^2}\left[\frac{z_m + jx_r}{z_r + z_m}\right]}{z_{bs}\frac{(z_m + jx_r)}{(z_r + z_m)} + \frac{r_r}{s(2-s)}} \qquad (39)$$

Let $$V_e = \frac{V_s}{A^2}\frac{(z_m + jx_r)}{(z_r + z_m)} \qquad (40)$$

$$Z = z_{bs}\frac{(z_m + jx_r)}{(z_r + z_m)} = r + jx \qquad (41)$$

and $$R = \frac{r_r}{s(2-s)} \qquad (42)$$

Then $I_m = \frac{V_s}{z_{io}} + \frac{V_e}{z + R}$ where R is the variable

$$= \frac{V_s}{z_{io}} + \frac{V_e}{r + jx + R} \qquad (43)$$

The first term of this equation is the net work current with the load impedance R = infinity. This current is shown as OA in fig. 7.6. Further the most important thing in this equation is that the fraction $\frac{V_e}{(R+r)+jx}$ represents a family of vector currents so long as V_e and x remain constant where $R+r$ or merely R take all possible values from minus infinity to plus infinity. This result is indicated in fig.7.6. the diameter of the circle being shown by the line AB. The total input current I_m is represented by the vector <u>od</u>, the point d being free to move along the circumference of the circle as the value of R or the load impedance is varied.

The point B for $R+r = 0$ cannot be realized unless R is made -ve For $R = 0$, the operating point would fall some-where in the upper part of the circumference such as at E. The total current vector <u>OE</u> corresponding to a value $R = 0$ is the total current with the receiver terminals short circuited.

<u>REFERENCES</u>: B7, B9, B10 and J22.

CHAPTER NO. 8

COMPARISON OF TWO THEORIES REVOLVING FIELD AND CROSS FIELD THEORIES.

The advantages which may be expected of a good motor theory are:

(i) Should present an accurate picture by means of which mathematical expressions for the current, torque etc., may be obtained.

(ii) Should furnish the simplest possible picture concerning the important phenomena consistent with the accuracy desired.

(iii) Should furnish means for readily considering the effects of minor phenomena.

(iv) Should present a picture readily susceptible to high-handed simplification for the determination of limiting conditions such as no load, standstill, balanced operation etc.

(v) The theory should preferably not change form or require~~d~~ a new mathematical analysis when slight normal modifications of structure are made.

(vi) Should preferably show the relation between the motor in hand and motors of other types.

(1) In the matter of accuracy, the revolving field theory

and the cross field theory are equivalent because both give correct expressions for current, torque etc.

The equivalence of the two theories, in this matter, has been conclusively proved in the chapter on equivalent circuit where the same circuit has been derived by the application of both the theories. Further it has been shown by P.L. Alger and Kimball that the pulsating torque of the single-phase motor can be explained by the use of both the theories. There is only one respect in which the two theories give different results. This one exception occurs when the secondary winding of a single phase machine has sufficient skin effect to make its effective resistance and reactance at line frequency appreciably different from their values with direct current. Since the actual current in any one secondary conductor consists of components of two different frequencies the two components encounter different impedances and so the effective voltage accross the conductor is not equal to the product of the total current by any definite value of impedance. The cross field treats the entire current in one axis as being of line frequency and so does not take into account the variation of impedance with frequency. On this account, the rotating field theory gives more accurate results for machines of this type.

In fact, it has been pointed out by L.V. Bewley that better numerical agreement between the two theories can be obtained by putting (comparing the two equivalent circuits

given in fig. 5.3 of chapter on "double revolving field theory) " and fig. No. 7.5 in the chapter on "Equivalent circuit and circle diagram").

$$\frac{r_r}{2s} + \frac{r_r}{2(2-s)} = \frac{R}{s(2-s)}$$

(Revolving field) (Cross field)

and substituting the value of R thus obtained as the referred rotor resistance, r_r, in the equivalent circuit given in figure 7.5. (Actually the equivalent circuits derived by two theories were shown to the same on the assumption that $r_r = r_r'$).

(2) For plain a.c. commutator motor, the cross-field theory is simpler but for induction motor, the revolving field theory is simpler.

(3) When minor phenomena such as fundamental stator iron loss or the effect of stator harmonics are to be considered, the revolving field theory appears to have real advantages. Since with the stator fundamental iron loss considered the equivalent circuit 1(a) and 1(b) can be replaced by 1(c) or 1(d) by inserting parallel resistances with each of the "Apparent impedances". The effect of stator harmonics can be easily determined, by first calculating for each harmonic its magnetising reactance, secondary reactance and secondary resistance. Using these a single phase equivalent circuit

may be formed for each harmonic and connected in series with the fundamental circuit as shown in fig. 2. (The fundamental leakage reactance should be reduced by an amount equal to the sum of the magnetising reactance).

It is difficult to see how the cross-field could so simply yield results with such minor phenomena.

(4) When high handed simplifications are made to study limiting conditions, such as standstill, synchronous running etc., the revolving field theory really demonstrates its superiority.

(a) At standstill, the forward and backward impedances are equal and circuit can be reduced to the form shown in 3(a) and if x_m is assumed very large, then it can be simplified to fig. 3(b).

(b) At synchronous running $s = 0$, and the forward -field secondary impedance is infinite and so the circuit can be reduced to the form shown in fig. 4(a) and since slip is 4 the back-ward secondary impedance is very low compared to $\frac{x_m}{2}$ and hence the circuit can be further simplified to 4(b).

(c) No load speed: When rotational iron and mechanical losses are neglected, it is merely necessary to equate the forward and backward field apparent resistances and solve for s. Assume that at no load, the voltages impressed on the forward and backward fields will be the

same as at synchronism

$$s_{NL} \frac{E_{fo}^2}{r_r/2} = I_o^2 r_{bo}$$

Then $$s_{NL} = \left(\frac{I_o}{E_{fo}} \right)^2 \frac{r_r}{2} r_{bo}$$

(d) In a similar way, such questions as, what would happen if the secondary resistance or reactance were zero and what would happen if $\frac{x_m}{2}$ were made very low, can be answered very readily the revolving field theory.

(5) Modification such as multiple cage rotor can be easily taken into account in the equivalent circuit and the performance predicated by the use of the revolving field theory. The modified equivalent circuit is shown in Fig. 5.

(6) The revolving is the more general of the two theories. If the two components of the revolving field are equal, we have a single phase system; if they are unequal we have an unbalanced polyphase system. If one component vanishes, we have a balanced polyphase system and if the angular velocity of the rotating field is equal to zero we have a d.c. system. In other words all phenomena of rotating electrical machines can be explained as special cases of the general theory based on two oppositely rotating magnetic fields.

<u>REFERENCES:</u> B9, J23 to J25.

CHAPTER NO. 9

ANALYSIS BY SYMMETRICAL COMPONENTS

1. Introduction:

The method of symmetrical components originally was developed by C.L. Fortescue in 1918, in connection with the analysis of symmetrically wound polyphase induction machines operating under unbalanced conditions. Since its original development, the method has proved to be of inestimatable value in the analysis of problems involving a.c. rotating machines under both steady and transient conditions and in connection with transmission lines and transformer banks under fault conditions.

In this chapter, a single phase motor with only one winding will be considered as an unbalanced 3 phase motor with one line cut off. This is shown to result in an equivalent circuit exactly similar to the one obtained by the use of double revolving field theory. An approximation to the equivalent circuit suggested by Wagner and Evans is also given.

Then an unbalanced two phase winding machine (which is the case of an actual single phase motor with a starting winding) will be analysed by the use of symmetrical components. It is only here that this method offers a powerful weapon in analyzing a single phase motor.

Simpler expressions for torque and currents including

starting values) result from this analysis. The original work of W.V. Lyon and Kingsley will be followed for this form of analysis.

2. Single phase motor considered as a 3 phase motor with one line opened.

A single phase induction motor which has a symmetrical polyphase rotor winding (such as the squirrel cage rotor) can be treated as a Y connected 3 phase motor, operating with one line terminal open, by using one-half the single phase resistance and reactance as the resistance and reactance of the 3 phase motor (per phase value). Hence the reactance phase used here are all half the actual value of the single phase motor.

Fig. 9.1 shows the conditions of operation with one line opened. The sequence currents and voltages can be derived as below:

$$I_1 = \frac{1}{3}(0 + aI - a^2I) = \frac{1}{3} I (a - a^2)$$

$$= \frac{1}{3} I j \sqrt{3} = \frac{jI}{\sqrt{3}} \qquad (a)$$

$$I_2 = \frac{1}{3}(0 + a^2I - aI) = \frac{1}{3}(a^2 - a)I$$

$$= \frac{-j\sqrt{3}}{3} I = -\frac{jI}{\sqrt{3}} \qquad (b)$$

Thus the currents in the two circuits are same but are opposite sense.

$$E_{s1} = \frac{1}{3}(0 + aE_{sb} + a^2E_{sc}).$$

$$E_{s2} = \frac{1}{3}(0 + a^2E_{sb} + aE_{sc}) \qquad (c)$$

$$\therefore E_{s1} - E_{s2} = \frac{1}{3}\left[E_{sb}\,(a-a^2) + E_{sc}(a^2-a)\right]$$

$$= \frac{1}{3}(a - a^2)\,(E_{sb} - E_{sc}) \qquad (d)$$

But $E_{sb} - E_{sc} = E$, the applied emf.

$$\therefore E_{s1} - E_{s2} = \frac{1}{3}\, j\sqrt{3}\,E = \frac{jE}{\sqrt{3}} \qquad (e)$$

The basic equations for sequence currents and voltages are (a), (b), and (e). These can be used in setting up an equivalent circuit, shown in figure 2.

It can be seen that this circuit is the same as that derived already by the use of the double revolving field theory. Hence the equation for torque, power output etc. given under the chapter on double revolving field theory (section) can be used.

Approximation suggested by Wagner and Evans.

In the stator the positive sequence & negative currents are equal in magnitude. The rotor currents referred to the stator are equal to the stator current minus the exciting currents.

Since for normal operation, the slip s for the positive sequence currents is small, the positive sequence exciting currents are large. Under similar conditions, the slip for the negative sequence currents is large $= (2-s) \simeq 2$. The negative sequence exciting currents are therefore small.

(Already the equivalent circuit shown in fig. 9.2, neglects the increase in rotor resistance when at a frequency of $f(2-s) \simeq 2f$ under operating conditions).

Hence the magnetising reactance of negative sequence is neglected. The magnetising reactance is approximated to be near the machine terminals. Hence the circuit reduces to that shown in fig. 3.

$$\therefore \quad \frac{r_r(1-s)}{2s} - r_r \frac{(1-s)}{2(2-s)} = r_r(1-s)\left(\frac{1}{s} - \frac{1}{2-s}\right)$$

$$= \frac{r_r(1-s)(2-s-s)}{2s(2-s)} = \frac{r_r(1-s)^2}{s(2-s)}$$

$$\text{Torque developed} = k_s \frac{r_r(1-s)^2}{(2-s)(1-s)}$$

$$= k \frac{r_r(1-s)}{s(2-s)} \quad \text{where } k \text{ is a constant} \qquad (f)$$

$$\text{Shaft output} = \frac{k\, r_r(1-s)^2}{s(2-s)} \qquad (g)$$

when $s = 1$, there is no torque developed.

3. Analysis (by symmetrical components) of unsymmetrical two phase machine:

The method of symmetrical components is a revolving field theory and has many features in common with the revolving field theory. The principal difference is that in a split-phase motor, there are two windings (in space quadrature) and so by the revolving field theory it has to be considered as two single-phase motors in the same frame with their windings in quadrature each component motor producing a forward and a backward revolving field. Thus four revolving fields have to be considered. When analysed by the method of symmetrical components, the currents in the two stator phases are resolved into two symmetrical set of components whose mmfs produce respectively a resultant forward and a resultant backward revolving field, and only these two fields need be considered.

Assumptions: Only fundamental fluxes will be taken into account. The air gap is assumed to be uniform. The rotor has a symmetrical polyphase circuit of either the cage or the wound type.

Stator M.M.Fs.

The schematic diagram of the stator windings m and s, separated in space by α electrical radians is shown in the figure 9.4.

Let the instantaneous currents in the two windings be

$i_m = \sqrt{2}/I_m/ \sin (wt + \varphi_m)$ (1)

$i_s = \sqrt{2}/I_s/\sin (wt + \varphi_m + \beta)$ (2)

where $/I_m/$ and $/I_s/$ = effective scalar values of the 2 currents.

φ_m = time phase angle of i_m with respect to zero time.

= time phase angle by which i_s leads i_m

The space fundamental mmf. H set up by these 2 windings at any distance x (fig) will be,

$$H = C (N_m i_m \sin x + N_s i_s \sin \overline{x + \alpha}) \quad (3)$$

where C = a constant.

N_s, N_m, = effective fundamental turns of 2 windings.

x = distance in electrical radians, as shown in figure.

Let $\frac{N_m}{N_s}$ = ratio of Effective No. of turns = k (4)

∴ Substituting the values from equations (1), (2) & (4) in (3) we have

$$H = \sqrt{2} C N_m \left[/I_m/ \sin x \sin (wt + \varphi_m) + \frac{/I_s/}{k} \sin(x + \alpha) \sin (wt + \varphi_m + \beta) \right)$$

$$\therefore H \frac{C N_m}{\sqrt{2}} \Big[/I_m/ \cos (x - wt - \varphi_m) - /I_m/ \cos (x + wt + \varphi_m)$$

$$+ \frac{/I_s/}{k} \cos (x - wt - \varphi_m + \alpha - \beta) + \frac{/I_s/}{k} \cos (x + wt + \varphi_m + \alpha + \beta) \quad (5)$$

The first and third terms of equation (5) are rotating fields in the positive direction of x while the second and 4th terms are negatively rotating fields.

When the two mmfs (NI) are equal, i.e. in our case when $/I_s/ = k/I_m/$, and $\alpha + \beta = 180^\circ$, ie., the sum of the time angle between the currents and the space angle between the windings is 180° the negatively rotating fields cancel and only positively rotating fields result.

Similarly when $/I_s/ = k/I_m/$ and $\alpha - \beta = 180^\circ$, the positively rotating fields cancel and only negatively rotating fields result. That is (a) only positively rotating fields result if

$$I_s = k\, I_m \angle (180 - \alpha) = - kI_m \angle - \alpha \qquad (6)$$

and (b) only negatively rotating fields result if

$$I_s = kI_m \angle (-180 + \alpha) = - kI_m \angle \alpha \qquad (7)$$

Symmetrical components of stator current:

In general, I_m and I_s will set up both positively and negatively rotating fields. However, these currents can be resolved into components I_{m1} and I_{m2} such that,

$$I_m = \overset{+\text{ ve seq.comp.}}{I_{m1}} + \overset{-\text{ve seq. comp.}}{I_{m2}} \qquad (8)$$

$$I_s = - k\, I_{m1} \angle - \alpha - kI_{m2} \angle \alpha \qquad (9)$$

In these equations, the components I_{m1} and $-kI_{m1}\angle-\alpha$ set up only positively rotating fields and components I_{m2} and $-kI_{m2}\angle\alpha$ set up only negatively rotating fields. Hence the currents I_{m1} and I_{m2} are called the positive and negative sequence components of I_m.

From now on we shall take the particular case of a single phase machine having $\alpha = 90^o$ and assume φ_m = the phase angle of i_m with respect to zero time $= 0$.

$$\text{Then } I_m = I_{m1} + I_{m2} \tag{10}$$

$$I_s = -kI_{m1}\angle-90^o - kI_{m2}\angle 90 \tag{11}$$

Solving these two equation simultaneously, we get

$$I_{m1} = \tfrac{1}{2}\left(I_m + \frac{I_s}{k}\angle-90\right) \tag{12}$$

$$I_{m2} = \tfrac{1}{2}\left(I_m + \frac{I_s}{k}\angle 90\right) \tag{13}$$

In the instantaneous form, the currents i_m and i_s are resolved into instantaneous positive sequence currents.

$$i_{m1} = \sqrt{2}\,/I_{m1}/\sin wt. \qquad i_{s1} = \sqrt{2}\,k/I_{m1}/\sin(wt + 90^o) \tag{14}$$

and instantaneous -ve sequence currents.

$$i_{m2} = \sqrt{2}/I_{m2}/\sin wt \qquad i_{s2} = \sqrt{2}\,k/I_{m2}/\sin(wt-90^o) \tag{15}$$

The resultant positively rotating mmf. H_1 set up by the positive sequence stator currents i_{m1} and i_{s1}

can be calculated by substituting the values of i_{m1} and i_{s1} for i_m and i_s in the equation.

$$H = C\left[N_m i_m \sin x + N_s i_s \sin(x + \alpha)\right]$$

$$= C\left[N_m i_m \sin x + N_s i_s (x + 90^o)\right]$$

$$= C\left[N_m \sqrt{2}/I_{m1}/\sin wt \sin x + \frac{N_m}{k}\sqrt{2}\, k/I_{m1}/\sin(wt + 90)\right.$$

$$\left.\sin(x + 90)\right]$$

$$= C\, N_m/I_{m1}/\sqrt{2}\left[\sin wt \sin x + \sin(wt + 90)\sin(x + 90)\right]$$

$$= C\, N_m/I_{m1}/\sqrt{2}\quad \sin wt \sin x \quad \cos wt \quad \cos x$$

$$= C\, N_m/I_{m1}/\sqrt{2}\quad \cos(x - wt) \qquad (16)$$

Let us now consider a <u>symmetrical</u> 2-phase stator winding of N_m effective turns per phase, the 2 windings being in space quadrature. The stator positive sequence currents in such machine are (assuming there is unsymmetry, in the currents)

$$i'_{m1} = \sqrt{2}/I'_{m1}/\sin wt \quad \text{and} \quad i'_{s1}$$

$$= \sqrt{2}\,/I'_{m1}/\sin(wt \quad 90) \qquad (17)$$

Substituting these values in the equation for stator mmf., we get the mmf. set up by the stator currents in a 2-phase motor as

$$H' = C\sqrt{2}/I'_{m1}/N_m \cos(x - wt) \qquad (18)$$

The equation given above for H' may be compared with equation (16). It can be readily seen that the unsymmetrical motor (here the single-phase motor with their dissimilar windings in space quadrature but with their currents not necessarily in time quadrature) will set up exactly the same stator mmf. (rotating in a positive direction) as an equivalent symmetrical 2-phase motor having N_m effective turns, if the positive sequence component of the stator current of the single phase motor is equal to the positive sequence current of the 2-phase motir, (i.e.)

$$/I_{m1}/ = /I'_{m1}/ \tag{19}$$

In an exactly similar manner it can be shown that the unsymmetrical motor (single phase motor here) will set up exactly the same negative sequence stator mmf. as an equivalent symmetrical 2-phase motor having N_m effective turns per stator phase if the negative sequence component of the stator current of the single phase motor is equal to the negative sequence current of the 2-phase motor

(i.e.), $/i_{m2}/ = /I_{m2}/$ (20)

Rotor currents:

So far as the internal reactions within the rotor are concerned, it makes no difference how the positive sequence and negative sequence mmfs are set up. That is, as viewed from the rotor it is impossible to tell whether the stator mmfs are due to the components currents in the

windings of the unsymmetrical stator windings, or due to the positive and negative sequence currents of an unsymmetrical 2-phase currents in a symmetrical winding, provided the symmetrical components of the equivalent 2-phase currents (single phase) are related to the symmetrical components of the 2 phase currents (2 phase).

Having seen this relation, it is but a few steps to analyse the machine completely by the use of the well-known equivalent circuits for a symmetrical 2 phase motor (of course, with unsymmetrical currents resolved into symmetrical components). The equivalent circuits for both the sequences are given in figure No. 9.5.

The rotor leakage and magnetising impedances shown here in these circuit are the actual referred values (to the stator winding of N_m) whereas in the double-revolving field theory it is just half the actual referred values.

$$\text{The positive sequence impedance} = z_1 = \frac{z_{r1} z_m}{z_{r1} + z_m} \tag{21}$$

$$\text{The -ve -do-} = z_2 = \frac{z_{r2} z_m}{z_{r2} + z_m} \tag{22}$$

Neglecting core loss (i.e. $r_m = 0$),

$$z_1 = \frac{(r_r/s)x_m^2 + jx_m\left[\left(\frac{r_r}{s}\right)^2 + x_r(x_r + x_m)\right]}{\left(\frac{r_r}{s}\right)^2 + (x_r + x_m)^2} \tag{23}$$

$$z_2 = \frac{\left[r_r'/(2-s)\right] x_m^2 + jx_m\left[\left(\frac{r_r'}{2-s}\right)^2 + x_r(x_r + x_m)\right]}{\left(\frac{r_r'}{2-s}\right)^2 + (x_r \quad x_m)^2} \tag{24}$$

The induced emfs E_{m1} and E_{m2} given in the equivalent circuits are:

$$E_{m1} = I_{m1} z_1 \cdot E_{m2} = I_{m2} z_2 \tag{25}$$

<u>Components of voltages</u>:

The voltage induced in the main winding is

$$E_m = E_{m1} \quad E_{m2} \tag{26}$$

In the winding s(starting winding) of the unsymmetrical motor, the voltage induced by the positive sequence field will be $1/k$ times the voltage induced in the main winding and will lead this voltage by the angle α, which is equal to 90^o in the single-phase motor under consideration. Similarly, the voltage induced in the s winding by the -ve sequence field will be $1/k$ times the voltage induced in the main winding and will lag this voltage by the angle $\alpha = 90^o$. Hence the voltage induced in the s winding will be

$$E_s = \frac{E_{m1}\angle 90}{k} + \frac{E_{m2}\angle -90^o}{k} \tag{27}$$

Solving these two equations Nos. 26 & 27 for

E_{m1} and E_{m2}, we obtain

$$E_{m1} = \frac{E_m - kE_s\angle 90^\circ}{2} . \qquad E_{m2} = \frac{E_m - kE_s\angle -90^\circ}{2} \qquad (28)$$

We can see that the resolution of voltages into symmetrical components is different from the resolution of currents - this is because of the dissymmetry of the stator windings.

COMPONENTS OF STATOR IMPEDANCES:

Let z_{ms} = the stator leakage impedance of the main winding.
and z_{ss} = the stator leakage impedance of the starting winding.

The voltage drops in the 2 windings due to these impedances will be :

$$v_m = I_m z_{ms} . \quad v_s = I_s z_{ss} \qquad (29)$$

Resolving these voltages into symmetrical components.

$$v_{m1} = \frac{v_m - kv_s\angle 90^\circ}{2} . \quad v_{m2} = \frac{v_m - kv_s\angle -90^\circ}{2} \qquad (29)$$

The positive sequence stator leakage reactance drop,

$$v_{m1} = \frac{I_m z_{ms} - kI_s z_{ss}\angle 90^\circ}{2}$$

$$= \tfrac{1}{2}\left\{\left[(I_{m1} + I_{m2})\right] z_{ms} - kz_{ss}\angle 90^\circ\left[-kI_{m1}\angle -90^\circ - kI_{m2}\angle 90^\circ\right]\right\}$$

$$= \tfrac{1}{2}\left[I_{m1}(z_{ms} + k^2 z_{ss}) + I_{m2}(z_{ms} + k^2 z_{ss}\angle 180^\circ)\right]$$

$$= I_{m1} z_o + I_{m2} z_{22} \qquad (31)$$

where $z_o = \frac{1}{2}(z_{ms} + k^2 z_{ss})$ (32)

$z_{22} = \frac{1}{2}\left[z_{ms} + k^2 z_{ss}\underline{/180}\right]$ (33)

Similarly proceeding, we can derive for v_{m2}

$v_{m2} = I_{m1} z_{21} + I_{m2} z_o$ (34)

where $z_{21} = \frac{1}{2}(z_{ms} + k^2 z_{ss}\underline{/-180})$ (35)

Equations for the single-phase motor:

Equating the positive sequence component of the applied potential to the sum of the positive sequence components of the induced voltages and positive sequence components of the voltage drops due to external impedances and stator leakage impedances.

The positive sequence applied potential

$= V_{m1} = I_{m1} z_1 \quad v_{m1}$ (36)

$V_{m1} = \frac{1}{2}(V_m - kV_s\underline{/90^\circ}) = I_{m1}(z_1 + z_o) + I_{m2} z_{22}$ (37)

Similarly equating for the negative sequence components,

$V_{m2} = \frac{1}{2}(V_m - kV_s\underline{/-90^\circ}) = I_{m1} z_{21} + I_{m2}(z_2 \quad z_o)$ (38)

Solving these two equations Nos. 37 and 38 for I_{m1} and I_{m2},

$$I_{m1} = \frac{1}{2}\frac{(V_m - kV_s\underline{/90^\circ})(z_2 + z_o) - (V_m - kV_s\underline{/-90^\circ})z_{22}}{(z_1 + z_o)(z_2 + z_o) - z_{21} z_{22}} \quad (39)$$

$$I_{m2} = \frac{1}{2}\;\frac{(V_m - kV_s\underline{/-90^\circ})(z_1 + z_o) - (V_m - kV_s\underline{/90^\circ})\; z_{21}}{(z_1 + z_o)(z_2 + z_o) - z_{21}\; z_{22}} \qquad (40)$$

The actual current in the m winding,

$$I_m = I_{m1} + I_{m2}$$

$$I_s = - kI_{m1}\underline{/-90} - kI_{m2}\underline{/90^\circ}$$

$$= k\; jI_{m1} - kjI_{m2}$$

$$= kj(I_{m1} - I_{m2}) \qquad (41)$$

The torque in synchronous watts, is then

$$T = 2\;(/I_{m1}/^2 r_1 - /I_{m2}/^2 r_2) \qquad (42)$$

where r_1 and r_2 are the resistance parts of the impedances z_1 and z_2 respectively.

Now substituting for z_o, z_{22} and z_{21} in terms of the winding impedances z_{ms} and z_{ss}, and putting $v_m = v_s$ (as it is so in a single phase machine), we have for the sequence currents,

$$I_{m1} = \frac{V}{2}\left[\frac{z_2 - jkz_{ms} - k(z_2 + jkz_{ss})\underline{/90^\circ}}{z_1 z_2 + (z_1 + z_2)\left(\frac{z_m + k^2 z_{ss}}{2}\right) + k^2\; z_{ms}\; z_{ss}}\right] \qquad (43)$$

$$I_{m2} = \frac{V}{2}\left[\frac{z_1 + jkz_{ms} - k(z_1 - jkz_{ss})\underline{/-90^\circ}}{z_1 z_2 + (z_1 + z_2)\;(z_m + k^2 z_s)/2 + k^2 z_{ms} z_{ss}}\right] \qquad (44)$$

At standstill, $s = 1$, and $z_1 = z_2 = z$ (say) then the expressions for I_{m1} and I_{m2} reduces to,

$$I_{m1} \text{ (Starting)} = \frac{V}{2}\left[\frac{z - jkz_{ms} - k(z + jkz_{ss})\angle 90}{(z + z_{ms})(z + k^2 z_{ss})}\right] \quad (45)$$

$$I_{m2} \text{ (Starting)} = \frac{V}{2}\left[\frac{z + jkz_{ms} - k(z - jkz_{ss})\angle -90}{(z + z_{ms})(z + k^2 z_{ss})}\right] \quad (46)$$

$T_{st} = 2\ r\left[/I_{m1}/^2 - /I_{m2}/^2\right]$ where r is the resistance part of $\underline{z}$ (47)

<u>REFERENCES</u>: B11, B12, J26 to J28.

CHAPTER NO. 10.

SPECIAL PROBLEMS OF SINGLE PHASE MOTORS

The uniqueness of the single-phase motor is evident in the absence of starting torque, in the double frequency pulsation of its running torque and its high starting current. Starting torque has been considered in detail in the second chapter; in this chapter the other two aspects will be considered.

At first the torque in motion is evolved from fundamentals followed by derivations for no load slip and slip for maximum torque. The pulsation of torque is then discussed in detail. The analysis by Mr. Alger and Mr. Kimball to explain this is taken up - only the revolving field theory is used to find expressions for the pulsating components of the torque; the analysis by the cross field theory is omitted here. Mr. Button's concept of Rotor Flux Locus helps in the visualisation of the presence of the pulsating torque; this is explained in short. Lastly Edward Bretch's explanation of the double frequency torque pulsation is given.

High starting current of the single phase motor results in the familiar flickering of lights connected to the same circuit due to the voltage dip caused by the high starting current. A new design by Mr. Williams to reduce voltage dip is also discussed in this chapter.

I. Average torque:

(a) Derivation of expression for torque while running:

It has been proved in the chapter on "Starting torque" that there is zero torque when the rotor is at rest. It is found that if the rotor is set in motion, it will continue to run; this means that there is resultant torque when the rotor is in motion.

Let the peripheral velocity of the rotor be v; then now x is a function of time, i.e. $x = vt$. Hence the equation No. (   ) of the chapter 2, can be written as:

$$\phi = \frac{2TL}{\pi} B_m \sin wt \cos \frac{\pi v}{T} t \qquad (1)$$

$$= \frac{TL\ B_m}{\pi} \sin (w - w_1)\ t + \sin (w + w_1) t \qquad (2)$$

where $$w_1 = \frac{\pi v}{T} \qquad (3)$$

The emf. induced in a coil is:

$$e = \frac{d\phi}{dt} 10^{-8}$$

$$= \frac{TL\ B_m}{\pi} \left[(w + w_1).\cos (w + w_1) t + (w - w_1).\cos (w - w_1) t\right].\ 10^{-8} \text{ volts.} \qquad \ldots (4)$$

It can be seen that the flux and emf. may now be considered as consisting of two components, one of frequency $(w + w_1)/2$ and the other of frequency $(w - w_1)/2$, both operating in the one coil.

The current is given by:

$$i = \frac{TL\ B_m .\ 10^{-8}}{\pi} \left\{ \frac{w + w_1}{\sqrt{r^2 + (w + w_1)^2 L^2}} \cos \left[(w + w_1)t - \varphi_1\right] \right.$$

$$\left. + \frac{w - w_1}{\sqrt{r^2 + (w - w_2)^2 L^2}} \cos \left[(w - w_1)\ t - \varphi_2\right] \right\} \qquad (5)$$

$$\text{where } \varphi_1 = \tan^{-1} \frac{(w \quad w_1)\ell}{r} \qquad \varphi_2 = \tan^{-1} \frac{(w - w_1)\ell}{r} \qquad (6)$$

$$i = \frac{TL\ B_m\ 10^{-8}}{\pi} \left\{ \sin \varphi_1 \cos \left[(w + w_1)t - \varphi_1\right] + \sin \varphi_2 \cdot \cos \left[(w - w_1)t - \varphi_2\right] \right\} \qquad (7)$$

It can again be seen that the current may be regarded as consisting of two components. Tangential pull is:

$$P = \frac{2LiB}{10} \text{ dynes} \qquad (8)$$

B can be shown to be (by putting $x = vt.$ in equation No. under the second chapter).

$$B = \frac{B_m}{2} \left[\cos (w - w_1)t - \cos (w + w_1)t\right] \qquad (9)$$

Substituting this value for B and the expression for i given in equation No.(7) in equation No. (8) we get,

$$P = \frac{TL^2B_m^2 10^{-9}}{L} \Big\{ \sin\theta_1 \cos\left[(w + w_1)t - \theta_1\right]$$

$$\sin\theta_2 \cos\left[(w - w_1)t - \theta_2\right]\Big\} . \left[\cos(w-w_1)t - \cos(w+w_1)t\right] \qquad \ldots \quad (10)$$

In finding the mean value of this expression it is only necessary to find the mean value of the products of terms having the same frequency. These products are:

$$- \sin\theta_1 \cos\left[(w+w_1)t - \theta_1\right] \cos(w + w_1)t$$

and $\quad \sin\theta_2 \cos\left[(w - w_1)t - \theta_2\right] \cos(w - w_1)t$

Their mean values over a cycle are respectively:

$$- \tfrac{1}{2} \sin\theta_1 \cos\theta_1 = - \tfrac{1}{4} \sin 2\theta_1$$

$$\tfrac{1}{2} \sin\theta_2 \cos\theta_2 = \tfrac{1}{4} \sin 2\theta_2$$

Hence $\quad P_{av} = \dfrac{TL^2 B_m^2 . 10^{-9}}{4\pi\ell} (\sin 2\theta_2 - \sin 2\theta_1) \qquad (11).$

If there are N coils on the rotor the value of P_{av} is N times the above expression. It is apparent that two torques act on the rotor of a single phase induction motor - (1) which may be termed forward component corresponding to θ_2 and (2) the backward component corresponding to θ_1. Under standstill conditions $w = 0$, $\theta_1 = \theta_2$ and the magnitudes of the backward and forward components are equal; hence the resultant torque is zero. If the rotor is set in motion

practical values of r and ℓ are such that $\sin 2\theta_2 > \sin 2\theta_1$ and a resultant torque is created. Assuming there is no external load on the motor when this torque becomes equal to that due to friction and windage, the rotor accelerates until $\sin 2\theta_2$ is in excess of $\sin 2\theta_1$ by an amount necessary to balance the friction and windage. It is evident that even in the absence of load and losses the motor can never attain synchronous speed. If synchronous speed were attained we should have $w_1 = \omega$, $\theta_2 = 0$ and the P_{av} would be negative quantity.

(b) <u>No load slip</u>:

It is evident that the performance of the single-phase induction motor is dependent on the values of r and . Expressing $\sin 2\theta_1$ and $\sin 2\theta_2$ interms of w, w_1, ℓ and r.

$$P_{av} = \frac{T v L^2 B_m^2 10^{-9}}{2\pi} \left[\frac{(w - w_1)}{r^2 + (w-w_1)^2 \ell^2} - \frac{(w + w_1)}{r^2 + (w+w_1)^2 \ell^2} \right] \qquad \ldots (12)$$

For

$$P_{av} = 0 \text{ either } w_1 = 0 \text{ or } w^2 - w_1^2 = \frac{r^2}{\ell^2}$$

$$\therefore \quad 1 - \frac{w_1^{\,2}}{w^2} = \frac{r^2}{w^2 \ell^2}$$

Under normal working conditions $w_1 \simeq w$

$$\therefore \quad 1 + \frac{w_1}{w} \simeq 2$$

But $1 - \frac{w_1}{w} = s = \text{slip}$

$$\therefore \quad s = \frac{r^2}{2w^2\ell^2} \tag{13}$$

This is the slip when the rotor is unloaded and free from losses, Hence in order to keep s small, the ratio r/w should be kept small.

(c) Slip at which the maximum torque occurs:

Differentiating the expression for P_{av} (equation No.) with respect to w_1 and equating to zero we have

$$- \frac{r^2 - (w - w_1)^2\ell^2}{\left[r^2 + (w-w_1)^2\ell^2\right]^2} - \frac{r^2 - (w + w_1)^2\ell^2}{\left[r^2 + (w + w_1)^2\ell^2\right]^2} = 0 \tag{14}$$

The first of these two terms is proportional to the rate of change of the forward component of the torque and the second to the backward component.

In the vicinity of the maximum of the forward component, the second rate of change is small compared with the first.

Multiplying the above equation by $\left[r^2 + (w - w_1)^2\ell^2\right]^2$ and denoting the resulting second term by - k, we have,

$$r^2 - (w - w_1)^2\ell^2 - k = 0$$

$$\therefore \quad r^2 - s^2 w^2\ell^2 - k = 0$$

$$\therefore s^2 (w^2 \ell^2) \quad k - r^2 = 0$$

$$\therefore swL = \pm \frac{\sqrt{4 (r^2 - k)}}{2}$$

$$\therefore s = \frac{\sqrt{r^2 - k}}{w \ell} \qquad (15)$$

If the backward component were negligible when the forward component is at its maximum.

Then $k = 0$ and therefore, $s = \dfrac{r}{w \ell}$ (16)

II. Torque Pulsation:

(a) On power flow in general:

The power input to a single phase circuit is necessarily of a pulsating character, as each time the incoming current or the voltage passes through zero, the power does likewise.

Let $\sqrt{2} V \sin wt$ = voltage impressed on a single phase circuit and $\sqrt{2}\, I \sin (wt - \varphi) =$ current in that circuit where φ is the angle of lag of current.

Then at any instant the power flowing in the line is

$$= \sqrt{2}\, V \sin wt.\ \sqrt{2}\, I \sin (wt - \varphi)$$

$$= 2\, V\, I \sin wt \sin wt - \varphi \quad = 2\, V\, I\, \tfrac{1}{2} (\cos\varphi - \overline{\cos 2\, wt - \quad})$$

$$= VI\, (\cos \varphi - \overline{\cos 2\, wt \; - \varphi})$$

Whatever the p.f. is, a single phase a.c. circuit always draws from the line a double frequency, alternating power equal to the full volt amps. The active power is the average power input and is, of course, proportional to the power factor. The maximum power flow is the sum of the active (or average) power, and the alternating power.

In a balanced polyphase circuit, each separate phase draws both active and alternating power as above described. But at each instant, the sum of the alternating powers of the phases is zero, so that the only net power flowing in the circuit is the active power, distributed equally among the phases. There is no net alternating power flowing in the lines, although each particular line carries an alternating power equal to its full volt amps.

If a current is represented as a jb and voltage as c jd, referred to the same reference axis then the product, (ac - bd) + j(bc + ad) is equal not to the (active power) + j(reactive power) as might be supposed but to the <u>single phase double frequency alternating power</u>. To obtain the true or active power by multiplication, it is necessary to first reverse the sign of the j term of either voltage or current, when the product becomes, (ac + bd) + j(bc - ad). The real term represents the active power VI cos α - and the j term the so-called reactive power V I sin α . The reason for this is: when the sign of one j term is reversed, the resulting vector product is at an angle equal to the difference

of the angles of the voltage and current and when the sign is not reversed, the angle of the vector product is equal to the sum of the voltage and current angles. As the angles in this vector representation are angular velocities times time (w t), the former is a zero frequency power (constant power) and the latter is a double frequency alternating power.

(b) Energy flow in a single phase induction motor:-

Since the power inflow to a single-phase motor is necessarily a pulsating one, whereas the power taken by the load is uniform the incoming energy must be alternately stored during the peaks and supplemented by released energy during the depressions. This energy is stored as Kinitic energy in the revolving parts, so that the torque exerted by the single phase motor is of a pulsating character corresponding to the electrical energy input and the rotor speed continually fluctuates electrical energy input and the rotor speed continually up and down in response to the torque variations as indicated in the figure No. 2

Further more, even at no load, and supposing the friction torque negligible, the motor torque pulsates at double frequency for the reason that the potential energy of the magnetic field in the cross axis is supplied entirely by the torque since there is no direct mutual induction between the main and cross axis. This magnetic energy is alternately stored and released twice in each cycle and so constitutes an important cause of torque (and speed) variations entirely

independent of the variations due to the load. The ratio between the double frequency torque due to the load and that due to the cross field, is proportional to the ratios of watts output to the magnetising volt amps. For very small single phase motors the no load torque variation is large and the additional variations due to the load are not very important, while for large machine the reverse is true.

The exact amount that the speed fluctuates must depend on the M.I. of the rotor and upon the restoring torque set up as a result of the departure of the speed from synchronism. If the speed varies 1 per cent from synchronous the flux will cut the rotor bars at a frequency of 1 per cent the line frequency; the slip frequency currents will flow in the rotor bars which will produce a torque (equal to the load torque which would normally produce 1 per cent slip) in such a direction as to restore the speed to synchronous. For the production of this double frequency resisting torque of such a load, a third harmonic must be drawn from the line and so the general conclusion is that all single phase motors must draw a certain amount of third harmonic current from the line, regardless of the existence of magnetic saturation. The amount of this third harmonic current should be greater, the less the normal slip of the motir, and the less the inertia of the rotor. It is believed that the third harmonic current of this character drawn by commercial single phase motors is of very small amplitude.

(c) Analysis by the Revolving field theory:

The forward component of flux cuts rotor conductors at slip frequency and induces in them useful currents similar to those in a polyphase induction motor secondary. The backward component of flux cuts the rotor conductors at a frequency (2-s) times line frequency and so indues in them large currents which practically neutralize the primary current and so reduce the net backward flux to a very small value.

As the two fields are produced by the same primary current, their circuits are connected in series and the equivalent circuit can be drawn as shown in Fig. 3. This has already indicated in the double revolving field theory.

From the equivalent circuit if I_m is the total current

$$I_f = \frac{j\, I_m x_m}{\frac{r_r}{s} + j\,(x_r + x_m)} \tag{17}$$

$$I_b = \frac{j\, I_m x_m}{\frac{r_r}{(2-s)} + j(x_r + x_m)} \tag{18}$$

$$e_f = j\,\frac{x_m}{2}\,(I_m - I_f) \tag{19}$$

$$e_b = j\,\frac{x_m}{2}\,(I_m - I_b) \tag{20}$$

$$\therefore\ e = e_f + e_b = j\,\frac{x_m}{2}\,(2I_m - I_f - I_b) \tag{21}$$

$$e = V - I_m(r_s + jx_s). \tag{22}$$

Since the forward and backward fields exist in a common magnetic structure, each flux makes a torque with both the currents I_f and I_b and hence there are in all four separate torques.

The torque produced by the forward revolving flux with I_f is of constant magnitude, independent of the time; so also the torque produced by the backward revolving flux with I_b. This is true since the two fluxes are of constant magnitude in time but moving in space. Thus the meaning of j in the torque equations to be derived hereunder is a rotation of 90° in space, instead of in time.

The principal torque is that due to the action of the forward revolving flux and I_f. The watts developed by this torque at synchronous speed are equal to:

$$\overset{*}{e}_f I_f = -j\frac{x_m}{2}(I_m - I_f)I_f \tag{23}$$

$$e_f = \frac{I_f}{2}\left(\frac{r_r}{s} + j x_r\right) \tag{24}$$

$$\overset{*}{e}_f I_f = \frac{I_f^2}{2}\left(\frac{r_r}{s} - j x_r\right) \tag{25}$$

Reducing the forward field branch of the equivalent circuit we get the single impedance of the value:

$$z_f = \frac{j\frac{x_m}{2}\left(\frac{r_r}{s} + j x_r\right)}{\frac{r_r}{s} + j(x_m + x_r)} \tag{26}$$

Similarly for the backward field,

$$z_b = \frac{j\frac{x_m}{2}\left(\frac{r_r}{2-s} + jx_r\right)}{\frac{r_r}{2-s} + j(x_m + x_r)} \tag{27}$$

$$e = (z_f + z_b)\, I_m \tag{28}$$

Substituting the values for z_f and z_b from equations (26) and (27) in equation (28), we get,

$$e = jI_m x_m \frac{r_r^2 + jr_r(x_m + 2x_r) - s(2-s)\, x_r(x_m + x_r)}{r_r^2 + 2jr_r(x_m + x_r) - s(2-s)(x_m + x_r)^2} \tag{29}$$

$$\therefore I_m = \frac{e}{jx_m} \frac{r_r^2 + 2jr_r(x_m + x_r) - s(2-s)(x_m + x_r)^2}{r_r^2 + jr_r(x_m + 2x_r) - s(2-s)x_r(x_m + x_r)} \tag{30}$$

$$e = V_s - I_m\,(r_s + jx_s) \tag{31}$$

$$T_{ff} = j(I_m - I_f)\frac{x_m}{2} I_f - \frac{I_f^2}{2}\left(\frac{r_r}{s} + jx_r\right) \tag{32}$$

Substituting the value for I_m from equation (30) in the equation (17) we obtain,

$$I_f = \frac{e\; jx_m\left[r_r^2 + 2jr_r(x_m + x_r) - s(2-s)(x_m + x_r)^2\right]}{j\, x_m\left[\frac{r_r}{s} + j(x_r + x_m)\right]\left[r_r^2 + jr_r(x_m + 2x_r) - s(2-s)x_r(x_m + x_r)\right]} \tag{33}$$

When the value of I_f from this equation is put into the equation (32) and the resulting equation [illegible] produced by the forward revolving flux with I_f (neglecting the j terms as they only measure the phase displacement in space, as already explained).

$$T_{ff} = \frac{e^2 r_r s \left[r_r^2 + (2-s)^2 (x_m + x_r)^2 \right]}{2 \left[r_r^4 + r_r^2 (x_m + 2x_r)^2 - 2s(2-s) r_r^2 x_r (x_m + x_r) + s^2 (2-s)^2 x_r^2 (x_m + x_r)^2 \right]} \qquad (34)$$

when $s = 1$; $T_{ff} = \dfrac{e^2 r_r}{2(r_r^2 + x_r^2)}$ (35)

Similarly proceeding, the torque produced by the backward revolving flux with I_b is

$$T_{bb} = \frac{e^2 r_r (2-s) \left[r_r^2 + s^2 (x_m + x_r)^2 \right]}{2 \left[r_r^4 + r_r^2 (x_m + 2x_r)^2 - 2s(2-s) r_r^2 x_r (x_m + x_r) + s^2 (2-s)^2 x_r^2 (x_m + x_r)^2 \right]} \qquad \ldots (36)$$

when $s = 1$;

$$T_{bb} = \frac{e^2 r_r}{2(r_r^2 + x_r^2)} \qquad (37)$$

This is exactly the same as that obtained for the forward field with I_f, at standstill (equation No. 35) showing that the backward and forward field torques are equal and opposite at standstill.

The net constant torque in synchronous watts is equal to the difference of (36) and (34) i.e.

$$T = T_{ff} - T_{bb}$$

$$= \frac{e^2 r_r(1-s)\left[s(2-s)(x_m+x_r)^2 - r_r^2\right]}{\left[r_r^4 + r_r^2(x_m+2x_r)^2 - 2s(2-s)r_r^2 x_r(x_m+x_r) + s^2(2-s)^2 x_r^2(x_m+x_r)^2\right]^2}$$

To calculate the alternating power, we must take the sum of the products of the backward field voltage by I_f and the forward field voltage by I_b. Since it is now only a question of alternating power, the phase relations of I_f and I_b with fluxes they act upon are of no importance. The voltage representing the forward flux is

$$e_f = j\frac{x_m}{2}(I_m - I_f) = \frac{I_f}{2}\left(\frac{r_r}{s} + jx_r\right) \tag{39}$$

and that representing backward field is

$$e_b = j\frac{x_m}{2}(I_m - I_b) = \frac{I_b}{2}\left(\frac{r_r}{2-s} + jx_r\right) \tag{40}$$

Since the relative speeds of I_f and e_b and of I_b and e_f are twice synchronous speed, the torques made by their interaction are double frequency torques. The synchronous double frequency torque is,

$$A = T_{fb} - T_{bf} = e_f I_b - e_b I_f \tag{41}$$

when the values for e_f and e_b from equations (39) and (40) are substituted we get

$$A = \frac{I_f\, I_b\,(1-s)r_r}{s(2-s)} \tag{42}$$

When I_f and I_b are substituted by values in terms of

e and impedances,

$$A = \frac{(1-s)^2 e \left[r_r \left[r_r^2 + 2jr_r(x_m + x_r) - s(2-s)(x_m + x_r)^2\right]\right]}{\left[r_r^2 + jr_r(x_m + x_r) - s(2-s)x_r(x_m + x_r)\right]^2} \quad (43)$$

$A = 0$ when $s = 1$. ie., there is no alternating torque component also when the rotor is at standstill.

(d) Explanation of pulsating torque by the Rotor flux Locus Concept of single phase induction motor by C.T.Button.

The reason for the pulsating and average motor torque components can be clearly visualized by the concept outlined herebelow:

Here the rotor is conceived of briefly as an inductive "Circularly symmetrical" rotating body which tends to maintain total flux constant in magnitude and direction with respect to itself. This inductive effect prevents total flux from disappearing at the instant the primary field is zero; and its resistance to change in direction of total flux through it produces torque.

This theory considers the total flux traversing the air gap and gives a complete picture of the variation in magnitude and direction of the total or resultant flux. Here the rotor is considered as a fixed one and the fluxes are referred to this axis.

Assumptions: Since no quantitative analysis is

contemplated stator resistance and leakage reactance are considered negligible.

A two pole motor is considered (Fig. 4) The motoring torque is assumed to be clockwise. The flux crossing the air gap in the axis of the stator winding poles will be called the primary flux.

Hence at synchronous speed, the axis of the primary flux will be rotating with an angular velocity of $2\pi f$ in a counter clockwise direction, with reference to the axis of reference fixed in relation the rotor.

Condition at synchronous speed - primary mmf.

1 If the rotor were at standstill, the flux locus would be a straight line, the instantaneous magnitude of the flux pulsating sinusoidally positive and negative with respect to time. At synchronous speed, this primary mmf. or primary flux vector would be a circle as shown in fig. 3. This may also be considered as representing the locus of flux through the rotor if there were no rotor conductors i.e. representing the primary flux as stated above. The max. primary flux is shown as F_M in fig. 5. The locus would be traversed in a counterclockwise direction by the vector twice per cycle; although its angular velocity is only $2\pi f$.

2. Effect produced by rotor conductors.

The rotor is an inductive body, circularly symmetrical.

The effect of rotor inductance (not the leakage reactance) will be to resistá any change in magnitude or direction of the total flux through it, i.e., due to change in magnitude and direction of the primary flux currents will flow in the rotor conductors producing a secondary flux at right angles to the primary flux and this flux will have a magnitude and direction tending to maintain the total resultant rotor flux constant at OF (See fig. 6). When the primary flux is zero the secondary flux will be a max. and perpendicular upward. This simply means that the cross field is max. when the main field is zero.

However, the inductive effect of the rotor will not be 100% in maintaining constant flux through the rotor at synchronous speed, due to the effect of rotor resistance and leakage reactance. The actual total rotor flux at synchronous speed in a practical motoris shown in fig. 7.

Normal motoring condition:

It is now but a step to the conception of the locus of total flux at any particular amount of slip. This is shown in fig. 8. where the flux vector progresses around the rotor in a clockwise sense (taking 2 steps forward and one step backward, so to say).

Torque is produced by change in direction of flux through the rotor. The instantaneous torque will be a function of flux magnitude and angular velocity (torque would not be directly proportional to angular velocity course).

Inspection of fig. 8, shows that at synchronous speed there is a pulsating torque with a net or average counter clockwise value. In other words, it requires power input to drive the motor at synchronous speed even with the losses neglected, i.e. the no load point is at some speed slightly below synchronism.

Inspection of fig. 8 reveals that with normal slip, there exists a pulsating double frequency torque component and also the average motor torque. The magnitude of the main flux will be the radius of the outer envelope of the total flux locus, and the quadrature flux will be represented by the radius of the inner envelope.

(e) A new concept for visulazing the reason for torque pulsation enuciated by Edward Bretch:

In synch-motors and generators it is usual to provide amortisseur windings. They are used (a) to prevent sudden shifting of flux across the pole faces as for example to prevent "hunting" and (b) to dampen out field-flux pulsations as for example to stablize the field of a synchronous generator against pulsating single-phase armature reactions.

In a single phase squirrel cage motor, at synchronism the primary field excitation sets up a flux in the rotor as well as in the stator. The squirrel cage which is moving in synchronism with the field excitation forms a most effective amortisseur winding by which induced currents

sustain the rotor flux substantially constant during the time the primary excitation goes through zero and the rotor moves from pole to pole. Thus these amortisseur currents sustain the rotor flux and the mechanical rotation of the rotor carries this sustained flux of the single phase motor.

Under running conditions, the rotor runs slightly below synchronism. Each time a rotor pole approaches a stator pole the primary magnetising force pulls the rotor field into synchronism and due to the same action as a polyphase motor tends to pull the rotor along with it, producing the torque. Thus the torque is a series of impulses one for each primary current impulse or of double frequency.

III. Design of single phase motors to minimize voltage Dips.

Common types of single phase fractional horse power motors have starting currents five to ten times their operating currents. The voltage dips and light flicker caused by frequency starting of these motors are often objectionable.

It has already been mentioned under the "Starting torque" chapter that Nema has set up standards which limit the starting current for fractional horse power motors. Now a new method proposed by Mr. J.E. Williams to reduce the starting currents (hence the voltage dips) will be discussed. The disadvantages of this new method are also mentioned at the end.

1. Relation between starting current and starting torque: Torque efficiency.

The reactive component of the starting current can be easily adjusted by using parallel starting capacitors; hence the power component of motor starting current is of primary importance.

The power input necessary to produce a given starting torque varies considerably with different types of induction motors. An ideal motor is one with a maximum ratio of torque to input power. A Balanced polyphase motor in which all losses are in the rotor is an ideal motor.

In the comparison of starting performance of various motors, a concept of torque efficiency is helpful. Torque efficiency may be defined as:

$$\text{Per cent torque efficiency} = \frac{\text{Torque (in synch-watts)} . 100}{\text{Total input power (in watts.)}}$$

Starting torque efficiencies in excess of 50% occur in many polyphase motor designs, but much poorer starting torque efficiencies are found in fractional horse power single phase motors (about 10% for resistance split phase motors upto 38% for some of the capacitor start motors).

The decrease in torque efficiency of single phase motor as compared with polyphase motors is due to the two factors:-

(a) Unbalance of copper between start and main windings - the large losses in the small starting coils (b) The imperfect phase-splitting. Deviations from perfect phase splitting

cause backward components of mmf and torque the effect of which is to subtract from the torque and add to the power input.

2. Design changes in improving the starting performance of Capacitor-start motor:

Starting torque efficiency is increased by (a) improved phase splitting (b) increased rotor resistance or (c) decreased stator resistance. More space is required for low resistance stator coils. Increased rotor resistance gives increased losses while operating under load. Sligh-tly improved phase splitting and higher starting torque efficiency occur with a larger capacitor - however this results in increased torque instead of the desired reduction of power input at the start.

The usual starting circuit is not suitable for the development of motors with reduced power input at the start. The main winding is designed by the operating requirements of the motor. The two possible adjustments are capacitor size and auxiliary winding turns. With standard starting circuit, the best torque efficiency is reached at a much higher torque than is required for a general purpose motor.

However, when the capacitor is connected in series with the main winding, adjustment of starting current in each of the two motor circuits is possible. The phasor diagram is shown in fig. (No.). The current in the main winding is determined by the size of the capacitor. The starting current in the auxiliary winding can be varied by changing the turns in that winding. Near perfect phase splitting and good

torque efficiency at start can be obtained for a wide range of torques by suitable adjustments of capacitor size and auxiliary winding turns.

3. <u>Disadvantages of this type of motor:</u>

(a) High cost.

(b) The main winding current is conducted through a motor switch contact all the time the motor is in operation- the probability of switch failure is greater than in the present capacitor start motor.

(c) The motor is completely disconnected from the line for a very short time during the operation of the motor switch. It is necessary that the starting switch operate to open the capacitor circuit before connecting the main winding directly across the line; otherwise the capacitor discharge current would weld the switch contacts. The transient current immediately following reconnection of the main winding to the power supply may be nearly as trouble-some as larger starting currents.

(d) The starting circuit is not practical for capacitor run motors.

<u>REFERENCES:</u> B6, J29 to J32.

CHAPTER NO. 11

STANDARDS

I. NEMA STANDARDS FOR FRACTIONAL H.P. SINGLE PHASE MOTORS.

The basis is to define motor output in terms of breakdown torque and to indicate by means of a service factor the maximum continuous output safely permissible within the limits of a reasonable temperature rise.

(1) Breakdown torque:

For the purpose of defining horsepower rating, the value of breakdown torque must fall within a specified range. The table below gives typical values for 50 cycles 4 pole motors in ounces-feet at an approximate full load speed of 1425 rpm. including for comparison figures the normal full load torque and breakdown values expressed as a percentage of full load values. The range includes the higher figure down to, but not including, the lower figure.

Table 1

H.P.	Torque in oz-ft. Full load	Break-down	Break down torque as % of F.L.T.
1/8	7.37	13.8 - 19.8	188 - 269
1/6	9.82	19.8 - 25.8	202 - 261
1/4	14.73	25.8 - 37.8	175 - 257
1/3	19.64	37.8 - 48.5	193 - 247
1/2	29.47	48.5 - 69.5	165 - 236
3/4	44.21	69.5 - 99.0	157 - 224

For any design the minimum figure obtained on test will determine the appropriate h.p. rating.

(2) Temperature Rise and Service Factor.

All open-type general purpose motors have a maximum temperature rise of 40° C under normal service conditions (These are defined as ambient air temp. not exceeding 40°C (104°F) and height above sea level not exceeding 3300 ft.) These small motors are designed to develop, in addition, a higher output with a temperature rise of 50°C at which maximum efficiency is normally obtainable. The actual increased output is given by means of a service factor stamped on the motor name plate. This is a number by which the normal rating should be multiplied to determine the maximum safe motor load. A table of such factors is given below:

Table 2

H.P.	1/8	1/6	1/4	1/3	1/2	3/4
Service Factor	1.4	1.35	1.35	1.35	1.25	1.25

These service factors are intended to be used only under normal service conditions, as defined above, and when the motor is operating on the voltage and frequency for which it was designed. Thus an American standard 1/3 H.P. motor will develop 0.45 h.p. continuously when running in air temperatures upto 104°F. The temperature rise, however, will increase from 40°C (or less) at normal output to not more than 50°C on continuous maximum rating.

(3) Starting current:

Maximum values of starting current, (the figures being taken with the rotor locked) are laid down. These values are given below for 230V motors, irrespective of frequency:-

Table 3

H.P.	1/6 or less	1/4	1/3	1/2	3/4
Locked amps.	10	11.5	15.5	22.5	30.5

(4) Code letters:

These are marked on the motor name-plate to show input in KVA per H.P. with the rotor locked, which enables the branch-circuit protective devices to be correctly adjusted. Typical values showing the appropriate ratings for fuses and settings for circuit breakers (are given below. The KVA given includes the lower figure upto, but not including the higher figure).

Table 4.

Code letter.	Locked K.V.A. per H.P.	Per cent of full-load current Fuse rating	Breaker setting
A	0 - 3.15	150	150
B-E	3.15- 5	250	200
F-R	5 & higher	300	250

It is possible too to calculate the maximum starting current required from the relationship:-

Starting amps: $\dfrac{\text{Locked KVA per h.p. x rated h.p. x 1000}}{\text{rated voltage.}}$

II. BRITISH STANDARDS:

Salient features from BS.170(1939) (with amendments issued on at 1944 April, 1947 and August 1956) (Electrical performance of fractional h.p. electric motors and generators with class A Insulation) are given below:

3. Open Machine: An open end-bracket machine is one having end-brackets of which the bearings form an integral part, and in which there is no restriction to ventilation other than that necessitated by good mechanical construction.

5. The limits of rated voltage for fractional h.p. machines.

For a.c. motors or generators.

Single-phase: 100 to 250 volts.

6. The standard frequency for a.c. machines shall be 50 cycles per second.

10. Preferred horsepowers and speed:

The preferred h.ps. recommended for all types of motors (other than Universal motors) for sizes of 1/8 bh.p. to 1 b.h.p. inclusive shall be:

1/8, 1/6, 1/4, 1/3, 1/2, 3/4 and 1.

and the preferred speed shall be approximately 1400 rpm.

NOTE: To obtain the best characteristics for universal motors for general purposes, the full load speed should preferably be not less than 600 rpm.

11. Continuous rating:

This is a statement of the operating limits assigned to the machine by the manufacturer, defining the load at which the machine may be operated for an unlimited period under the conditions specified on the rating plate and in clause 37 while complying with the requirements of this specification.

17. The temperature-rise (limits of)

Table 5.

Part of machine	Temperature-rise measured by thermometer.
1. Windings insulated with class A material and cores with which they are in contact.	50°C
2. Commutator & slip rings	55°C
3. Uninsulated parts including cores not in contact with insulated windings.	The temp. rise shall in no case reach such a value that there is risk of injury to any insulating material or adjacent parts.

18. Method of measuring temperature: The temperature of the machine shall be measured by thermometers applied to the hottest accessible surfaces of the stationary parts of the machine during the test period, and by any other thermometers applied to the accessible surfaces of the rotating

parts as soon as the machine is stopped after the test.

19. Excess torque for motors -

(a) Motors with continuous rating, including totally enclosed motors:-

(b) All motors other than motors coupled to propeller fans and/or centrifugal loads shall be subjected to 25% excess overload in torque for 5 minutes.

22. High-voltage tests:-

The high-voltage tests in accordance with table shown below, shall be applied only to a new and completed machine in normal working condition with all its parts in place and unless otherwise agreed, shall the carried out at the manufacturer's works, preferably at the conclusion of the temperature test of the machine.

Table 6

Description of machine	Test voltage r.m.s.
(1) For all machines would for 50 volts or less.	500 volts.
(2) For machines would for voltages higher than 50 volts upto and including 250 volts.	1000 volts.

23. A high-voltage test shall be applied between the windings and the frame of the machine. The duration of the test shall be not less then 5 seconds for machines of continuous ratings upto and including 1/3 h.p. per 1000 rpm. or 250 watts or

VA per 1000 rpm. and not less than 1 minute for larger machines. The test voltage shall be at any frequency between 25 and 100 cycles.

24. Insulation resistance: shall not be less than one Megohms at 500 volts D.C.

37. The standard ratings given in this specification are suitable for machines operating under the following conditions of cooling air temperature and altitude:-

(a) A temperature of cooling air not exceeding 40°C.

(b) An altitude not exceeding 3300 ft. above sea level.

III. APPROXIMATE FULL LOAD CURRENTS OF SINGLE PHASE MOTORS:

Having seen the American and British standards it may be of interest to have an idea of the full load currents of single phase motors. Approximate values are given in the following table:

Table 7

H.P.	Currents in amps. 110 volts	Currents in amps. 220 volts.
1/8	2.7	1.35
1/6	3.0	1.50
1/4	4.0	2.0
1/3	5.0	2.5
1/2	7.5	3.75
3/4	10.5	5.25
1	13.0	6.50
1½	18.0	9.00
2	23.0	11.50
3	32.0	16.00
5	50.0	25.00
7½	70.0	35.00
10	94.0	47.00

REFERENCES: B4, B13, J33 and J34.

CHAPTER NO. 12

DESIGN

Main Dimensions:

For fractional horse power motors it is more satisfactory to use output in watts than in h.p., in the output equation.

$$C = \frac{D^2Ln}{\text{Watts output}} \qquad (1)$$

where C = output constant given in graph (Fig. 12.1)

D = the gap diameter.

L = core length

n = rpm.

Watts output = 373

$$\frac{\text{watts}}{\text{RPM}} = \frac{373}{1500} = .248$$

C, the output constant read from graph in figure 12.1 against .248 = 210 (2)

$$D^2L = \frac{373 \times 210}{1500} = 14 \times 3.73 = 52.2$$

The available stator punching is shown in figure No. 12.2 For this D = 3.5 inches. (3)

$$\therefore L = \frac{52.2}{3.5^2} = 4.25 \text{ inches..} \qquad (4)$$

Main Winding:

w_{ts} = tooth width = $\frac{9''}{64}$ Stackening factor = .95

Total tooth section = S_{ts} = $w_{ts} \cdot S_s \cdot \ell \times .95$ (5)

$$= \frac{9}{64} \times 28 \times 4.25 \times .95$$

= 16 sq. inch.

For a tooth density, of 10,500 lines per square inch, the total flux,

$$\phi_t = B_{ts}\, S_{ts} = 10{,}500 \times 16 = 1.68 \times 10^6 \text{ lines per sq. inch.} \qquad (6)$$

Flux per pole, assuming sine wave flux distribution in air gap,

$$\phi = \frac{E\ 45 \times 10^6}{f\ N\ C_w} \text{ lines} \qquad (7)$$

where E is the induced voltage and can in most cases be assumed equal to $0.95\ E_t$ for fractional h.p. single-phase induction motors.

The total flux $\phi_t = \frac{p\ \phi}{f_d}$

$$= \frac{p \times 0.95\ E_t\ 45 \times 10^6}{f\ N_m C_w\ f_d} \text{ lines} \qquad (8)$$

where f_d, the flux distribution constant, is equal to 0.637 for sine-wave flux distribution.

The winding constant C_w cannot be calculated until the distribution of the winding is known. For the usual winding, the factor C_w will lie between 0.75 and 0.85.

Taking C_w = 0.795, the total number of conductors in series for the main winding, using the equation (8)

$$N_m = \frac{P\, E_t \times 0.95 \times 45 \times 10^6}{f\, \phi_t\, C_w\, f_d}$$

$$= \frac{4 \times 230 \times 0.95 \times 45 \times 10^6}{50 \times 1.68 \times 10^6 \times .795 \times .637}$$

$$= 925.$$

Making a provisional number of turns per pole as 120, we get for the total number of conductors as 120 x 8 = 960

... (9)

The stator windings of single phase induction motors are generally of the concentric type. Proper winding distribution to produce sine-wave field will reduce harmonics of low order so that their effect will be small; the procedure to design the winding distribution is as indicated below. (The effects of harmonics have been dealt with in the 3rd & 4th Chapters).

Since there are 28 slots, slots per pole $= \frac{28}{4} = 7$.

Therefore the concentric winding is arranged as shown in the figure , the turns being distributed according to the sine law. The turns required in each coil can then be found as follows:

Coil 3 - 5 sin of ½ coil span $= \sin \frac{2}{7} \times 90 = .435$

Coil 2 - 6 sin of ½ coil span $= \sin \frac{4}{7} \times 90 = .785$

Coil 1 - 7 sin of ½ coil span $= \sin \frac{6}{7} \times 90 = .975$

$$\overline{2.195}$$

Percent turns per pole in coil 3-5 $= \frac{.435}{2.195} \times 100 = 19.8$

" " " " 2-6 $= \frac{.785}{2.195} \times 100 = 35.8$

" " " " 1-7 $= \frac{.975}{2.195} \times 100 = 44.4$

$$\overline{100}$$

$\therefore$ The actual number of turns in coil 3-5 $= .198 \times 120 = 24$

" " " " 2-6 $= .358 \times 120 = 43$

" " " " 1-7 $= .44 \times 120 = 53$

$$\overline{120} \qquad \ldots . \quad (10)$$

The winding distribution factor for concentric type single phase windings is a weighted mean chord factor or pitch factor and is calculated by multiplying the chord factor of each coil per pole group by the turns in the coil and dividing the sum of these products by the total number of turns.

There fore the winding factor for our machine is,

$$C_w = \frac{(.435 \times 24) + (.785 \times 43) + (.975 \times 53)}{120}$$

$$= \frac{10.4 + 33.8 + 51.5}{120}$$

$$= .8$$

The complete winding diagram is shown separately. We have taken a tooth flux density of 10,500 and $C_w = .795$ and obtained the value for N_m as 925. Now for the $N_m = 960$, and $C_w = .8$, The total flux in the to tooth,

$$\phi_t = \frac{1.68 \times 10^6 \times 925 \times .795}{960 \times .8} = 1.6 \times 10^6 \text{ lines} \quad (12)$$

$\therefore$ Flux density in the tooth $= B_{ts} = \dfrac{1.6 \times 10^6}{1.68 \times 10^6} \times 10500$

= 10000 lines per square inch. (13)

The flux per pole $= \dfrac{1.6 \times 10^6 \times .637}{4}$ = 256,000 lines... (14)

Yoke width = 3/8"

The stator yoke density $= \dfrac{256,000}{\frac{3}{8} \times 2 \times .95 \times 4.25}$

= 85,000 lines per sq. inch.... (15)

Now in order to determine the suitable size of the conductor let us proceed to find the input current.

Assuming that efficiency x power factor $= .5$ (16)

$$I_m = \frac{.5 \times 746}{230 \times .5} = 3.25 \text{ amps.} \qquad (17)$$

The current density in the stator conductor is made as high as temperature or efficiency guarantees will permit. It depends upon whether the motor is open or enclosed. For open-type motors (split phase, capacitor and repulsion start) the current density can usually be from 2500 to 3000 amps per square inch.

For a current density of 2800 amps per sq. inch. and a one-circuit winding, the section area of the stator conductor for main winding,

$$\frac{3.25}{2800} = .00116 \text{ sq.inch.}$$

No. 19 S.W.G. conductor has an area of 0.001257 square inch. For this conductor, then the current density is,

$$\frac{3.25}{.001257} = 2590 \text{ amps. per sq. in.} \qquad (18)$$

The slot area for the stampings chosen,

$$= \left(\frac{.65 + .9}{2}\right) 1.35 + (.1 \times .15)$$

$$= 1.06 \text{ sq.cm.}$$

$$= .164 \text{ sq. in.} \qquad (19)$$

Only the coils in 1-7 will occupy slots which will certain only main winding . Other slots, 3-5, 2 & 6 will have both the main and starting windings. The actual number of turns in coil 1-7 is, as shown in equation (10), 53.

$$\text{Hence the space factor} = \frac{53 \times .001257}{.164}$$

$$= .406 \tag{20}$$

Adopting super enamelled wire this space factor can be achieved.

Air Gap Length:

The length of the air-gap for fractional horse power motors can be determined approximately by the following empirical equation,

$$g = 0.005 + 0.00035D + 0.001\ell + 0.003v/1000 \tag{21}$$

$$= 0.005 + (0.00035 \times 3.5) + (0.001 \times 4.25) + (.003 \times 1.37)$$

$$= .01457 \text{ inch.}$$

Rotor bars:

The outside diameter of the rotor is therefore

$$= 3.5 - 2 \times .01457$$

$$= 3.471 \text{ inch.} \tag{22}$$

The rotor punching selected is shown in figure No. 12.3

This has 20 slots, which with the 28 stator slots can meet all the requirements for a quite motor as discussed under the last section of the chapter and "Noise in single phase induction motor"

$$\text{Rotor bar area} = \frac{5}{12} \times \frac{5}{16} = .049 \text{ sq.in.} \qquad (23)$$

<u>End rings</u>:

The distribution of the currents in the bars and the end rings of a squirrel cage winding is shown in Fig. No. It is evident from the figure that the current in each bar divides in the end-ring, one-half returning through a bar a pole pitch to the right and the other half through a bar a pole pitch to the left. If the maximum value of the current in each bar is $\sqrt{2}\, I_r$ and if the current is maximum in all bars bars at the same time, then the maximum value of the current in the end ring

$$= \frac{\sqrt{2}\; I_r}{2} \times \frac{N_r}{2p}$$

where N_r = number of rotor bars.

However, the current in all the bars do not reach their maximum at the same time; at a particular instant the current distribution in space is sinusoidal as shown in the figure No. 12.4.

Then the max. value of the current in the end ring (with respect to time) $\neq$

$$= \frac{\sqrt{2}\ I_r\ \ N_r}{2\ \ \ 2p} \times \frac{2}{\pi}$$

$\therefore$ R.M.S. value of current in the end ring

$$= \frac{1}{\sqrt{2}}\ \frac{\sqrt{2}\ I_r\ \ N_r}{2\ \ \ 2p} \times \frac{2}{\pi}$$

$$= \frac{1}{\pi}\ \frac{I_r\ N_r}{2p} \qquad (24)$$

Usually the end ring is designed for the current density as in the rotor bar.

$\therefore$ Area of cross section of the end ring

$$= \frac{N_r \times .049}{\pi \times 2p}$$

$$= \frac{20 \times .049}{\pi \times 4}$$

$$= .078 \qquad (25)$$

The end ring section is $= \frac{5}{16}$ inch x $\frac{1}{4}$ inch.

The outer diameter of the ring $= 3.471 - 2 \times \frac{5}{16}$

$= 2.846$ inch (26)

Inner diameter $= 2.846 - .5 = 2.346$ inch (27)

The end ring is shown in figure No. 12.5

Rotor Flux Densities:

Minimum width of the rotor tooth $= \left(\frac{\pi \times 2.846}{20}\right) - \frac{5}{12}$

$= .29$

The rotor length is made as $= 4.5$ inch (28)

The flux in the rotor is less than the flux in the stator because of the stator leakage flux. This leakage flux factor can be taken as $= 0.95$.

$\therefore$ Rotor tooth density $= \dfrac{\phi_t \times 0.95}{.29 \times 4.5 \times 20 \times 0.93}$

where 0.93 in the denominator is the stacking factor for the rotor stampings.

ϕ_t (from equation 16) $= 1.6 \times 10^6$

$\therefore$ Rotor tooth density $= \dfrac{1.6 \times 10^6 \times 0.95}{.29 \times 4.5 \times 20 \times 0.93}$

$= 63,000$ lines per sq. in. (29)

Rotor core radial depth $= \dfrac{5''}{8}$

$\therefore$ Rotor yoke density $= \dfrac{256,000 \times .95 \times 9}{2 \times \frac{5}{8} \times 4.5 \times .93}$

$= 42,500$ lines per sq. in. (30)

Air Gap density:

$$\text{Air gap density} = \frac{1.6 \times 10^6 \times .95}{\pi \times 3.471 \times 4.25}$$

$$= 32{,}600 \text{ lines per sq. in.} \qquad (31)$$

Stator Resistance:

Length of the half-mean turn for each of the coils per pole of a concentric winding

$$= \frac{4.2\,(D + ds)}{S_s} \times \text{slots spanned} + \ell \qquad (32)$$

= crossed portion shown in figure No. 12.6.

4.2 is used instead of to take into account the excess portion overhanging from the stator.

$$\text{For coil 3-5;} \left\{ \frac{42(3.5 + .53)}{28} \times 2 + 4.25 \right\} 24 = 131$$

$$\text{For coil 2-6;} \left\{ \frac{4.2(3.5 + .53)}{28} \times 4 + 4.25 \right\} 43 = 287$$

$$\text{For coil 1-7;} \left\{ \frac{4.2(3.5 + .53)}{28} \times 6 + 4.25 \right\} 53 = 417$$

$$835$$

$$L_{sm} = \text{Length of half-mean turn} = \frac{835}{120} = 7 \text{ inches} \qquad (33)$$

The resistance of the 19 s.w.g. Cu. at 25 degrees = 19.439 per 1000 yds.

∴ The resistance of the main winding at 75 degrees,

$$r_{sm} = \frac{7 \times 960}{3 \times 12} \times \frac{19.439}{1000} \times 1.2$$

$$= 4.35 \text{ ohms.} \quad (34)$$

$$\text{Resistance at } 25^0 = \frac{4.35}{1.2} = 3.62 \text{ ohms.} \quad (35)$$

<u>Rotor Resistance:</u>

The total resistance of the squirrel cage bars

$$= \frac{\ell_b \, N_r \, r}{10^6 \, a_b} \text{ ohms} \quad (36)$$

where ℓ_b = length of each bar.

a_b = the are of section of bar.

$r \times 10^{-6}$ = the resistance of cu. per square inch.

The resistance of the two end rings

$$= \frac{2\pi D_{er} \, r}{10^6 \, a_{er}} \text{ ohms} \quad (37)$$

where D_{er} and a_{er} are the mean diameter and sectional area of the end ring.

The total resistance of the squirrel cage is calculated as equal to the total copper less divided by the $(\text{current})^2$, since the ratio of currents in end ring to that of the bar (as already shown in equation No. 24), is $\frac{N_r}{\pi (2p)}$, the total resistance of the cage winding is,

$$= \frac{\ell_b N_r r}{a_b 10^6} + \left[\frac{N_r^2}{\pi^2 (2p)^2} \frac{2\pi D_{er} r}{a_{er} 10^6} \right]$$

$$= \frac{N_r^2 r}{10^6} \left[\frac{\ell_b r}{a_b N_r} + \frac{.64 D_{er}}{(2p)^2 a_{er}} \right] \text{ ohms} \qquad (38)$$

The rotor resistance must be expressed in terms of the stator winding before it can be added to the stator resistance to give the total resistance of the motor. At standstill, the induction motor is simply a polyphase transformer; the equivalent resistance of the rotor is therefore equal to the total rotor resistance times the square of the ratio of the effective stator turns to the rotor turns. The number of phases in a squirrel cage winding is equal to the number of bars per pole $= \frac{N_r}{(2p)}$ and the number of turns in series per phase is equal to the number of pole pairs $= p$.

The total resistance of a squirrel cage winding in terms of the stator winding is then,

$$= \left[\frac{(\frac{N_m}{2}) \ k_p k_d \ m}{p \ \frac{N_r}{2p}} \right]^2 \frac{r}{10^6} N_r^2 \left[\frac{\ell_b}{a_b N_r} + \frac{0.64 D_{er}}{(2p)^2 a_{er}} \right]$$

$$= \frac{N_m^2 k_p^2 k_d^2 m^2 r}{10^6} \left[\frac{\ell_b}{a_b N_r} + \frac{0.64 D_{er}}{(2p)^2 a_{er}} \right] \qquad (39)$$

where, m, number of phases = 2 for single phase machines.

When the radial width of the end-ring is large, as is

often in small motors, the end ring resistance must be corrected to take into account the effect of the non-uniform current distribution in the ring. The constant by which the end ring resistance should be multiplied to take into account this effect, has been evolved by Mr. P.H. Trickey in his paper "Induction motor resistance Ring width" in Electrical engineering vol. 55, Feb. 1936 page 144. This constant k_{ring} is given in the graph (Fig. No. 12.7) for various values of ratio of inside to outside diameter of the end-ring (for different poles). Therefore the equivalent rotor resistance per phase is

$$\therefore \quad r_{rm} = \frac{N_m^2 \, k_p^2 \, m \, r}{10^6} \left[\frac{\ell_b}{a_b N_r} + \frac{0.64}{(2p)^2} \frac{D_{er}}{a_{er}} k_{ring} \right] \text{ohms.} \qquad \ldots\ldots (40)$$

No skewing of rotor bars is proposed.

Length of rotor = 4.25 + 0.25 = 4.5 inches.

ℓ_b = Length of bar = 4.5 + 0.625 + 0.25 = 5.5 inches.

Ratio of inside to outside diameter of end ring = $\frac{2.346}{2.846}$

= 0.825

From graph in Fig. No. 12.7 K_{ring} = 0.95

$\therefore$ Rotor resistance in terms of stator main winding.

$$r_{rm} = \frac{N_m^2 \, C_{wm}^2 \times m \times r}{10^6} \left[\frac{\ell_b}{N_r a_b} + \frac{0.64}{(2p)^2} \frac{D_{er}}{a_{er}} k_{ring} \right] \text{ohms.}$$

$$= \frac{960^2(.8)^2 \times 2 \times .692}{10^6} \quad \frac{5.5}{20 \times .049} \quad \frac{0.64 \times 2.595}{4^2 \times \frac{5}{64}} \times .95$$

$$= 5.5 \text{ ohms at 25 deg. C.} \qquad (41).$$

$$= 5.5 \times 1.2 = 6.6 \text{ ohms at 75 deg. C...} \qquad (42)$$

Reactances:

The method of calculating the leakage reactance is in general the same as used for polyphase machines; some changes, must however, be made because of the different type of slot and winding used on single phase induction motors. Practical design experience, it is stated, has shown that the leakage reactance equations below give sufficiently close results for the design of most fractional horsepower induction motors.

$$X_{slot} = 2\pi f N_m^2 C_{wm}^2 10^{-8} \frac{6.38\,L}{Q_1} F_{ss} \frac{Q_1}{Q_2} F_{sr} \quad \text{ohms} \qquad (43)$$

where F_{ss} and F_{sr} are calculated from figures No. 12.8 and 12.9.

The zigzag reactance for stator and rotor in terms of main winding,

$$X_z = 2\pi f N_m^2 C_{wm}^2 10^{-8} \frac{2.13L}{Q_1 \ell_g} \frac{(w_{ts} + w_{tr})^2}{4(t_{1s} + t_{1r})} \quad \text{ohms} \qquad (43)$$

where w_{ts} = width of stator tooth at air-gap surface.

w_{tr} = width of rotor tooth at air-gap surface.

t_{1s} = stator tooth pitch at air gap surface.

t_{1r} = rotor tooth pitch at air gap surface.

The end connection leakage reactance,

$$X_e = 2\pi f N_m^2 C_{wm}^2 10^{-8} \frac{\pi (D + d_s)\text{av. coil span}}{Q_1 \ (2p)} \text{ ohms} . \quad . \qquad (45)$$

Since there is no skew $X_{skew} = 0$ (46)

In the three reactance formulae, the common term will be first evaluated.

$$\text{The common term} = A = 2\pi f \ N_m^2 \ C_{wm}^2 \ 10^{-8}$$

$$= 2\pi \times 50 \times 960^2 (.8)^2 \times 10^{-8}$$

$$= 1.85 \qquad (47)$$

For stator slot: $\frac{W_{s2}}{W_{s3}} = \frac{.65}{.9} = .72$

(given in fig. 12.10)

From the graph of fig. 12.8 for this ratio, $\emptyset = 0.46$

$$\therefore F_{ss} = (.46 \times \frac{1.35}{.9}) + (\frac{0.1}{0.15}) + 0$$

$$= .69 + .667$$

$$= 1.357$$

For rotor slot (given in fig. 12.11): $\frac{d_1}{W_{s3}} = \frac{.5}{.4} = 1.25$

$$\frac{W_{s2}}{W_{s3}} = 1$$

Now referring to graph given in figure No. 12.9 for the above two values,

$$\emptyset = .55$$

$$\therefore F_{sr} = .55 + 0 + \frac{2 \times 0.125}{(0.4 + 0.4)}$$

$= .862$

Substituting values in equation (43), we get

$$\therefore X_{slot} = 1.85 \left[\frac{6.38 \times 4.25}{28}\right]\left[1.357 + \frac{28}{20} \times .862\right]$$

$$= 4.44 \text{ ohms} \qquad (48)$$

$$\text{Stator slot pitch} = t_{1s} = \frac{\pi \times 3.5}{28} = 0.392 \text{ inch.}$$

$$\text{Rotor slot pitch} = t_{1r} = \frac{\pi \times 3.5}{20} = 0.55 \text{ inch.}$$

Further $w_{ts} = 0.392 - 0.0625$

$= 0.3295$ inch.

$w_{tr} = t_{1r} = 0.55$ inch.

Substituting in equation No. 44,

$$\therefore X_z = 1.85 \quad \frac{2.13 \times 4.25}{28 \times .0145} \left[\frac{(.3295 + .55)^2}{4(.392 + .55)}\right]$$

$$= 8.5 \text{ ohms} \qquad (49)$$

The end connection leakage reactance,

$$X_e = 1.85 \quad \frac{\pi \times 4.095 \times 4}{28 \times 4}$$

$$= .85 \text{ ohms.} \qquad (50)$$

x_{1m} = Total leakage reactance $= 4.44 + 8.5 + .85$

$= 13.8$ ohms (51)

The magnetising reactance can be calculated from the formula

$$x_m = 2\pi f\, N_m^2\, C_{wm}^2\, 10^{-8}\; \frac{0.645\, \ell\, T}{\ell_g\, k\, (2p)\, F_{sf}} \quad \text{ohms} \qquad (52)$$

where k is the air-gap coefficient $= k_s \times k_r$, to take into account the fringing effect due to the presence of slots.

$$k_s = \frac{t_{1s}}{w_{ts} + y\,\ell_g}$$ where y is read from curve in fig. 12.12. No. against the ratio $\dfrac{\text{stator slot opening.}}{\text{air gap length.}}$

$$k_r = \frac{t_{1r}}{w_{tr} + y\,\ell_g}$$ where y is read from curve in fig. 12.12 against the ratio $\dfrac{\text{rotor slot opening}}{\text{air gap length.}}$

F_{sf} is the saturation factor, which is the ratio of the total ampere-turns for the magnetic circuit to the air-gap ampere-turns only, is difficult to predetermine from direct-current magnetisation curves. The shape of the magnetizing current wave is not sinusoidal because of the non-linear nature of the magnetisation curve of the core material. By use of alternating current magnetisation curves the magnetisation characteristics can be fairly accurately calculated. For single phase induction motors the saturation factor will usually be between the limits 1.10 and 1.35. When the magnetic densities in the teeth and yoke are low, the low value applies and when high a higher value must be used

$$T = \text{pole pitch} = \frac{\pi D}{2p}$$

For stator slot the coefficient k_s is calculated as follows:

$$\text{The ratio} \quad \frac{\text{slot opening}}{\text{air gap length}} = \frac{.0625}{.0145} = 4.3$$

$\therefore$ y for this ratio from graph in fig. No. 12.12 = 2.13

$$\therefore \quad k_s = \frac{.394}{.3295 \quad (2.13 \times .0145)} = \frac{.394}{.3604} = 1.1$$

$$k_r = 1$$

$\therefore$ k = air gap coefficient = 1.1

The saturation factor F_{sf} is taken as = 1.26.

$$T = \frac{\pi \times 3.5}{4} = 2.75 \text{ inches.}$$

$$\therefore \quad x_m = 1.85 \times \frac{.645 \times 4.25 \times 2.75}{.0145 \times 1.26 \times 4 \times 1.1}$$

$$= 169 \text{ ohms} \tag{53}$$

The open circuit reactance, the reactance of the stator main winding with secondary open,

$$x_o = x_m + \frac{x_{\ell m}}{2} \quad ..$$

$$= 169 + 6.9 = 175.9 \text{ ohms} \tag{54}$$

$$\text{Leakage flux factor } k_r = \frac{x_o - x_{\ell m}}{175.9}$$

$$= \frac{175.9 - 13.8}{175.9} = .922 \tag{55}$$

$$k_p = \sqrt{k_r} = \sqrt{.922} = .96 \qquad (56)$$

Core loss:

The losses in the cores of the induction motors consist of the hysteresis and eddy current losses in the teeth and yokes due to the fundamental frequency plus additional losses. The additional losses comprise surface losses in the teeth due to variations in the air gap density, tooth pulsation losses due to variations in the tooth density, losses due to slot filing, losses due to non-uniform flux distribution, and losses in the end plates and end backets. In the stator core, the frequency of the flux reversals is equal to the line frequency; in the rotor it is equal to line frequency times the per cent slip. The loss in the stator teeth due to the fundamental frequency flux is equal to the loss per pound per cycle for the stator tooth density times the frequency of the flux reversals times the weight of the iron in the teeth. The loss per pound per cycle for various flux densities and for several grades of sheet steel is given by curves obtained from tests or samples in accordance with the American Society for Testing Materials. The loss in the stator yoke due to the fundamental frequency flux is calculated as explained for the teeth.

The additional losses are difficult to calculate. The surface losses in the teeth and the tooth pulsation losses

can be calculated by the method proposed by T. Spooner and I.F. Kinard. The total core losses for induction motors are generally 1.5 to 2.5 times the sum of the stator tooth and yoke losses due to the fundamental frequency flux. The multiplying factor should be obtained from tests of motors of similar design; when such data are not available 1.75 to 2.2 may be used.

$$\text{Volume of stator teeth} = \left[.394 \times \frac{1}{16} + \frac{9}{64} + \frac{17}{132} \right] \times$$

$$28 \times 4.25 \times .93$$

$$= .0246 \quad .0745 \quad 28 \times 4.25 \times .93$$

$$= 11 \text{ Cu inches.}$$

$$\therefore \text{wt} = 11 \times .278 = 3.06 \text{ lbs.}$$

Loss per lb. for tooth density 10300 is $5 \times .8 = 4$ watts.

$$\text{Area of c.s. of stator stamping} = \frac{\pi}{4} \left[(5 \tfrac{7}{16})^2 - 4.7^2 \right]$$

$$= 5.9$$

$$\text{Core volume} = 5.9 \times 4.25 \times .93$$

$$\text{wt} = 5.9 \times 4.25 \times .93 \times .278 = 6.5$$

core loss for density $86,800 = 3.5 \times .8$

core loss in yoke $= 3.5 \times .8 \times 6.5 = 23$ watts.

$$\text{Total core loss} = 18 + 12.24 = 30.24 \tag{57}$$

Taking additional losses into account, total core loss

$$= 30.24 \times 2 = 60.48 \tag{58}$$

$$\text{Friction \& windage} = \frac{4.5}{100} \times \tfrac{1}{2} \times 746 = 12.6 \qquad (59)$$

$$\text{Total no-load loss} = 12.6 + 60.48 = 73.08 \qquad (60)$$

Auxiliary winding:

For the capacitor start motor, the capacitor is relied upon to produce the phase displacement between the current in the main and auxiliary winding. The auxiliary winding circuit is opened after the rotor has attained approximately 75% of normal speed. The starting torque can be calculated from the formula

$$T_s = k_r \frac{1.88\, pE^2 k\, r_{rm}}{f} \left[\frac{r_a x_m - r_m(x_{\ell a} - x_c)}{(r_m + x_{\ell m})^2 (r_a^2 + x_{\ell a} - x_c)^2} \right] \text{ oz.ft.} \ldots \quad (61)$$

where r_a is the auxiliary winding resistance plus the resistance of the capacitor. For maximum starting torque, the value of x_c can be found by differentiating this equation with respect to x_c and equating to zero, which gives for x_c,

$$x_c = x_{\ell a} + \frac{r_a}{r_m} (z_m - x_{\ell m}) \text{ ohms.} \qquad (62)$$

The value of x_c for maximum starting torque for given main winding constant varies directly with $x_{\ell a}$ and r_a which in turn vary direct with the square of k, the ratio

of effective auxiliary winding turns to main winding turns.

For maximum cost of capacitor k should be large, but a large value of k increases the cost of the auxiliary winding. Hence a combination must be arrived at which gives the required starting characteristics at reasonable cost.

Choosing $k = 1.2$ and the winding constant for the auxiliary circuit $C_{wa} = .85$

The number of conductors in the auxiliary winding is,

$$N_a = k \frac{N_m C_{wm}}{C_{wa}} = \frac{1.2 \times 960 \times .8}{.85}$$

$$= 1080 \qquad (63)$$

$\therefore$ No. of conductors per pole $\frac{1080}{4} = 270$

For sinusoidal distribution the percentage of turns in coils,

Coil 6-9 - $\sin \frac{3}{7} 90 = .625 \qquad \frac{.625 \times 100}{2.525} = 24.8\%$

Coil 5-10 - $\sin \frac{5}{7} 90 = .9 \qquad - \qquad = 35.6$

Coil 4-11 $\sin \frac{7}{7} 90 = 1.0 \qquad - \qquad = 39.6$

$ \qquad 2.525 \qquad\qquad 100\%$

Actual No. of turns

6 - 9	= .248 x 135	- 33
5 - 10	-= .356 x 135	- 48
4 - 11	= .396 x 135	- 54
		135

(64)

$$\text{Winding constant} = \frac{(.625 \times 33) + (.9 \times 48) + (1 \times 54)}{135}$$

$$= .875 \qquad (65)$$

The final ratio of effective conductors,

$$\therefore \quad k = \frac{.875 \times 135}{.8 \times 120} = 1.23 \qquad (66)$$

Starting torque and current were calculated for several conductor sizes and gauge No. 23 was chosen finally.

Length of half mean turn

$$= \frac{\left[(.605 \times 3) + 4.25\right] 33 + \left[.605 \times 5 + 4.25\right] 48 \left[.605 \times 7 + 4.25\right] 54}{135}$$

$$= \frac{200 + 349 + 458}{135} = \frac{1007}{135} = 7.45'' \qquad (67)$$

Total length of auxiliary winding

$$= \frac{7.45 \times 135 \times 8}{36} = 223 \text{ yds.}$$

Resistance per 1000 yds = 53.998

$$\therefore \quad \text{Resistance of auxiliary winding} = \frac{53.998}{1000} \times 223$$

$$= 12.04$$

A.C. resistance = 12.04 x 1.1 = 13.25

Total main winding resistance = 4.35 + 6.6 = 10.95

Locked rotor resistance in term of auxiliary winding

$= r_{ra} = 1.23^2 \times 6.6 = 10$ (68)

$r_a = 13.25 + 10 = 23.25$ (69)

Total leakage reactance in term of aux. winding

$x_{\ell a} = 1.23^2 \times 13.8 = 20.8$ (70)

Main winding locked rotor impedance.

$Z_m = \sqrt{13.80^2 + 10.95^2} = 17.6$ (71)

Locked rotor current in main winding $= \dfrac{230}{17.6} = 13.08$

Capacitor reactance required for max. starting

$$x_c = x_{\ell a} + \frac{r_a}{r_m} (z_m - x_{\ell m})$$

$$= 20.8 + \frac{23.25}{10.95} (17.6 - 13.79)$$

$$= 28.9 \qquad (72)$$

$\therefore$ Capacitor in Micro F. $= C = \dfrac{10^6}{100\pi \times 28.9}$

$= 110$ Micro F.

Capacitor of 100 Micro F. is selected

Then $x_c = 31.82$ (73)

$z_c = \sqrt{23.25^2 + (31.82 - 20.8)^2} = 25.7$ (74)

$$I_{sa} = \frac{230}{25.7} = 8.95$$

$$\text{ct. density in aux. winding} = \frac{8.95}{.00045239}$$

$$= 19800 \text{ amps per sq. inch} \qquad (75)$$

<u>Starting torque:</u>

$$= \frac{.93 \times 1.88 \times 4 \times 230^2 \times 1.23 \times 6.6\left[23.25 \times 13.79 - 10.95\,(20.8-31.82)\right]}{(10.95^2 + 13.79^2)\left[23.25^2 + (20.8 - 31.82)^2\right]}$$

$$= 6.46 \times 10^4 \left(\frac{542}{310 \times 660}\right) \times .93 = 100 \text{ oz.ft.} \qquad (76)$$

Ratio of starts to full load torque = 3·6. (77)

$$\text{Wt. of Cu for main winding} = \frac{186}{1000} \times 14.573$$

$$= 2.71 \text{ lbs.} \qquad (78)$$

$$\text{Wt. of Cu for Auxiliary winding} = \frac{223}{1000} \times 5.2318$$

$$= 1.165 \text{ lbs.} \qquad (79)$$

Single phase Induction Motor Design sheet

H.P. ½ S.rpm - 1500 Cycles - 50, poles - 4, volts 230, Amps - 3.25

	Stator	Rotor		Stator	Rotor
Outside diameter	5,7/16"	3.471"	Tooth face	5/16"	.545"
Inside diameter	3.5"	0.75"	Tooth width	9/64"	.29"
Length	4.25"	4.59"	Depth below slot	3/8"	5/8"
No. of slots	.28	20			
Tooth pitch	.395"	.545"	Total Cu. section	.0665"	.049"

Slot No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Stator Main	53	43	24	-	24	43	53	53	43	24	-	24	43	54
Stator Auxiliary	-	33	48	54 -54	48	33	-	-	33	48	54 -54	48	33	-

Contd....

Winding	Main	Aux.	Rotor	Details
Conductor size	19 S.W.G. enamelled	23 S.W.G. enamelled	Barsection	5/32" x 5/16"
Conductor area	.0012569	.0004524	Length	5.47"
Amps. per sq. inch.	2590 (running)	19800 (starting)	Skew bars.	nil
			Material	Copper
Half mean time	7"	7.45"		
Conductor in series	960	1080	End ring section	5/16" x $\frac{1}{4}$"
C_W	0.8	0.875	Dia. outside	2.846
			Material	copper
Weight	2.71 lbs.	1.165lbs.	Dia. inside	2.346

Gap length	.0145"	Gap coeff - 1.14	Pole pitch - 2.75
Total flux	1.60×10^6	f_d - 1.637	Flux per pole - $.256 \times 10^6$

contd . .

Magnetic circuit details

	Section	Density	weight	Core loss
Stator teeth	2.48	103,00	3.06	12.24
Stator yoke	2.96	86500	6.5	18.0
Rotor teeth	4.05	63000		30.24
Rotor yoke	5.5	46500		Total core loss
				= 30.24 x 2
Air gap	7.6	33600		= 60.48 W

Machine constants

r_{sm} = 435	r_{sa} = 13.25	x_{em} = 13.8	x_{ea} = 20.8
r_{rm} = 6.6	r_{ra} = 10	z_m = 17.8	z_a = 31.2
r_m = 10.95	r_a = 23.25	k = 1.23	x_c = 31.82

	Full load						Torque		Locked
	amps.	Watts	effcy.	p.f.	App.efft.	rpm.	max.	start	amps.
Calculated									
Test									

CHAPTER - 13

PREDETERMINATION OF PERFORMANCE AND CONSTRUCTION.

Predetermination:-

By using the design constants arrived at in the previous chapter, the performance of the machine was predetermined by (1) Morrill's apparent impedance method

and (2) Veinott's method.

As explained in the chapter on revolving field theory, the per unit "apparent impedances" are read from the graphs given in figures 13.1 and 13.2. These are used to forecast the performance of the machine. The procedure adopted for the calculations and the results are given in Tables Nos. 13.1 and 13.2.

The second method is by the application of cross field theory as suggested by Mr. Veinott.

Constants that are used in this method of predetermination are first derived from the design data as given hereunder:

$\frac{1}{2}$ h.p. 230V 1500 r.p.m. (say)

Reactance $x_{lm} = 13.79$ $\quad x_o = 175.9$

r_{sm} at 75^o $= 4.35$

r_{rm} at " $= 6.6$

$k_p = .96$; $\frac{r_{rm}}{x_{lm}} = .478$

$$\frac{r_{rm}}{x_o} = .0375$$

$$I_m = \frac{E}{x_o} = 1.315$$

$$I_m r_{rm} = 8.67$$

$$F_1 = (2 - k_p^2) r_{rm} = 7.12$$

$$F_2 = (2\ r_{sm} + r_{rm}) \frac{r_{rm}}{x_o} = .574$$

$$F_3 = I_m \frac{r_{rm}}{x_o} = .367$$

$$F_4 = I_m r_{rm}\, x_r = 17.34$$

$$F_5 = I_m r_{rm}\, k_p = 8.33$$

$$F_6 = F_5^2\, r_{rm} = 456$$

$$F_7 = V k_p = 220$$

$$F_8 = (V k_p)^2\, r_{rm} = 321{,}000$$

$$F_9 = \frac{\text{Core loss (m)}}{E}$$

$$= \frac{30.24}{230}$$

$$= .1315$$

Core loss (m) = 30.24

Core loss (c) = 30.24

Friction & windage = 12.6

The actual performance calculations and the results are given in Table Nos 13.3 & 13.4.

The performance curves as determined by these two methods are shown in the next chapter along with the curves as per test results. (Figs. 14.4 and 14.5).

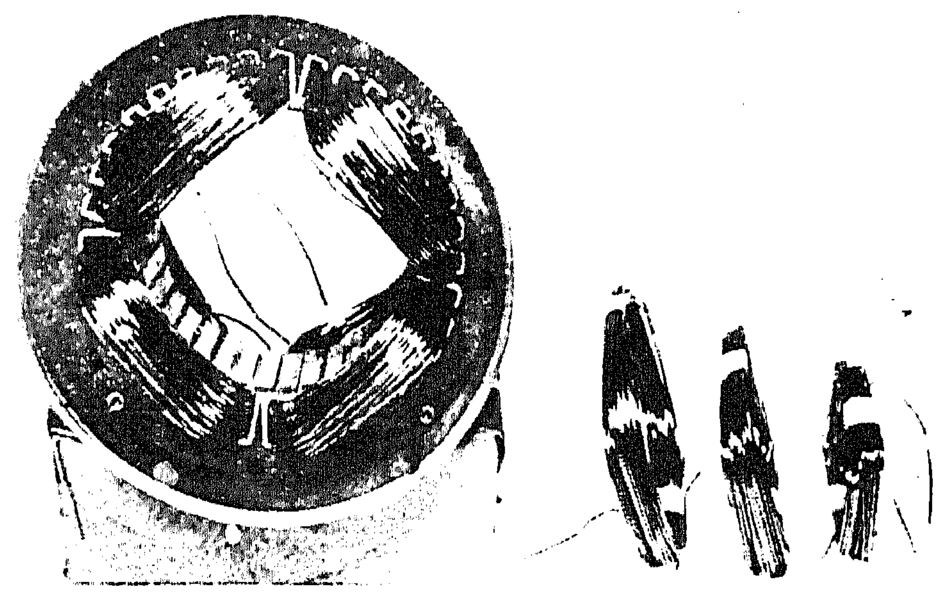
Construction.

(a) Method of construction adopted.

The actual constructional method adopted in making the motor will be given in brief. The machine has been constructed with the cast iron frame and end-covers.

The stator frame had been bored to receive stator stampings. This frame and end covers are shown in figures 13.3 and 13.4.

Then the windings are wound on the stator. The detailed winding diagram is given in Fig. No. 13.5. As explained under the previous chapter, the concentric winding method is adopted. A typical concentric winding is shown in Fig.13.5(a). No mould was used; the windings were hand wound and then inserted into the slots. Then the usual impregnation of the coils is done.



[redacted] Mould-winding.

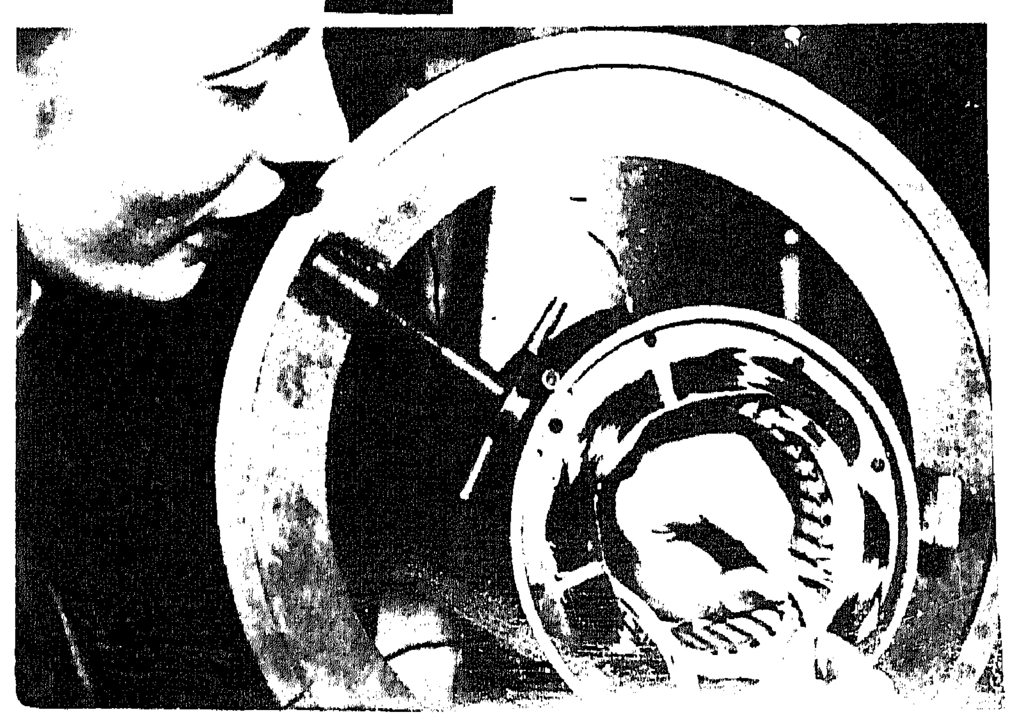


Fig 13.5(a)

Meanwhile the rotor shaft was got ready, after fixing its correct dimensions to receive the rotor stampings, fans, and bearings. Fans were fabricated. Eight fins which were cut from 1/16" mild steel sheet were welded to a common ring (made of 1/16" sheet) on the outer side and to a boss on the inner side to make a fan. One fan and one of the grease cups turned are given in Fig. No. 13.6. The end ring has already been shown in Fig. No. 12.5. An assembled view of the rotor is in Fig. No. 13.7.

Complete assembly of the machine is shown in Fig. No. 13.8.

(b) Fabricated motors welding construction Method.

For small motors, this method of construction is preferred. This is described in detail hereunder.

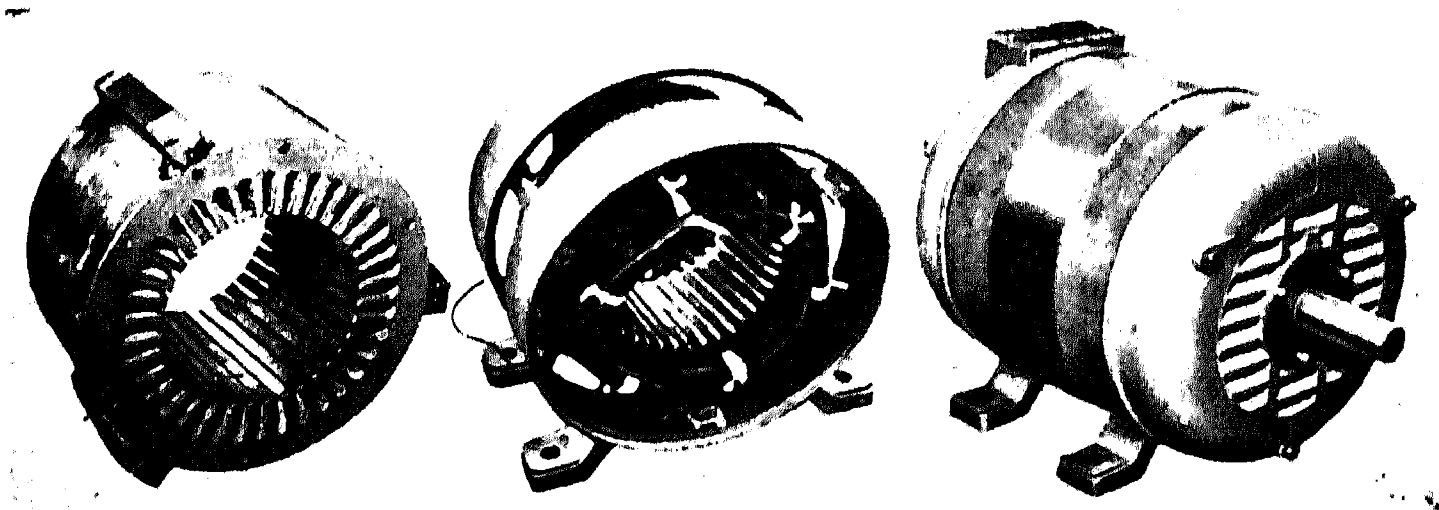
A fabricated 2 h.p. squirrel cage motor weighs only 54 lbs. as compared with over 100 lb for a machine of similar rating with cast iron frame and end covers.

The usual frame, bored to receive stator laminations is dispensed with. The laminations pack has welded to its periphery 3 mild steel bars on to which are subsequently welded two m.s. rings, spigots being turned in them to receive the die-cast aluminium end covers. The centre cover is perforated. Apart from reducing weight, this method of construction permits efficient cooling. Another feature of importance is robustness - a completed stator assembly of this type can be dropped through distances of 2 or 3 ft without the slightest damage being caused, not even broken feet.

The first stage of manufacture consists of blanking the stator and rotor laminations from electrical sheet steel by means of two blanking and piercing presses and a battery of notching presses. The laminations then go to the inspection department where, after having been passed, they are wired together in packs, each containing the right number for a particular machine, the number being checked by weight against a standard pack. The next operation is welding of the three rectangular section spacer bars on to the outside diameter of the assembled laminations and for this purpose a specially constructed jig is employed. When the

bars are in position a run of weld is made between them to give additional strength to the assembly. In this construction, instead of skewing the rotor slots, as is normal practice to reduce noise, the stator slots are skewed to achieve the same effect. This is done because it is more convenient to the method of construction and is easily arranged by suitably shaping the welding fixture.

The assembled laminations are then ready to receive the stator rings, which are held in position by means of another jig fixture and welded to the spacer bars. To reduce distortion of the rings during the welding operation a bevel is provided at each end of the spacer bars and a steel wedge is inserted between it and the rings. After cooling, the stator assembly is shot blasted to remove any rough edges and rust prior to the insertion of the coils.



Three stages in the assembly of fabricated motors showing (left) "welded up" lamination pack (centre) wound stator assembly, and (right) completed drip-proof machine ready for despatch

Fig 13.9.

In the winding shop, the stator coils are wound concentrically in groups of three from one length of wire, thus avoiding soldered joints. Insulating sheet is then placed in the slots and the coils inserted. After an insulation test, the leads to the terminal box are fitted. The final process is the vacuum impregnation of the coils. When this is done the stator are ready to receive the rotors and end covers.

The rotor laminations are prepared in batches each containing the correct number ~~of~~ in the same manner as those of the stator . By means of a press and jigs, they are firmly fixed in position, two "V" grooves in the raised edges forming a key to prevent rotary displacement. The larger machines have a small ring placed at each end of the laminations and welded into position to prevent longitudinal movement of the rotor.

Rotor bars are then dropped into place and brazed to a copper ring at each end, after which the outside diameter of the laminations is ground and treated with an anti-rust telley. At this stage the final assembly takes place, the fans and end covers being placed in position and the bearings adjusted.

Before despatch from works, each machine is finally checked in the electrical test department, where it undergoes a dynamometer test at no load and full load and is stalled six times.

TABLE NO. 13.1.

Predetermination of performance by the Apparent impedance method.

Motor constants.

$r_s = 4.35$; $x_s = 12.27$; $x_m = 169$; $w_i = 70.5$

$r_r = 6.6$; $x_r = 1.52$; $w_f = 12.5$ $\frac{x_r}{x_m} = .009.$

	Performance when s =	0	2.7%	3.0%	3.4%	4%	5%
1	RPM = (1-s) Syn. rpm.	1500	1460	1455	1449	1440	1225
2	$(x_m s)/r_r$	0	.692	.77	.87	1.02	1.23
3	R_{rf}	0	.48	.485	.495	.495	.479
4	X_{rf}	1	.648	.61	.56	.49	.379
5	$\frac{x_m(2-s)}{r_r}$	51.2	50.5	50.4	50.3	50.25	50
6	R_{rb}	.018	.0182	.018	.0185	.0190	.0195
7	X_{rb}	.01	.01	.012	.0125	.015	.0175
8	$R = (R_{rf}+R_{rb})x_m + r_s$	7.39	12.79	12.85	12.95	13.03	12.80
9	$X = (X_{rf}+X_{rb})x_m + x_s$	183.27	123,2	122.27	108.77	97.77	79.7
10	R/X	.074	.103	.104	.12	.133	.166
11	Cos Φ	.08	.125	.130	.15	.156	.2
12	$I = (V \cos \Phi)/R$	2.2	2.25	2.3	2.32	2.76	3.6
13	$W_o = (R_{rf}-R_{rb})x_m I^2 (1-s) - W_f$	0	230	300	320	380	460
14	$WI_f = EI \cos \Phi + W_i$	100	380	440	490	550	650
15	Efficiency = $\frac{\text{Output}}{\text{input}}$	0	60.5	68	66.5	69.5	70
16	R.P.M. = (1-s) Syn.rpm.	1500	1460	1455	1449	1440	1425
17	Torque	0	15	25	30	35	40
18	P.F.	.198	·65	·70	·75	·77	·78

Table No. 13.2

Predetermination by Apparent impedance Method.

Output in watts.	in % of F.L.	Input in watts.	Current amps.	slip	Torque	Efficiency	Power factor.
0	0	100	2.2	0	0	0	19.8
230	62	380	2.25	2.7	15	60.5	65
300	81	440	2.3	3.0	25	68	70
320	86.5	490	2.32	3.4	30	66.5	75
380	102.5	550	2.76	4.0	35	69.5	77
460	124.0	650	3.6	5.0	40	70	78

TABLE NO. 13.3

Predetermination by Veinott's Method.

1. S = rpm/s rpm.	.966	.973	.97	.95	.96
2. S^2	.9325	.9473	.94	.902	.92
3. $(1-s^2)$	.0675	.0527	.06	.098	.08
4. $(1-S^2)\ r_{sm}$	.2935	.229	.261	.426	.343
5. F_1	7.12	7.12	7.12	7.12	7.12
6. $U = (4)+(5)$	7.4135	7.349	7.381	7.546	7.468
7. $(1-S^2)x_{1m}$	.93	.727	.827	1.35	1.103
8. F_2	.574	.574	.574	.574	.574
9. $W = (7)-(8)$	.356	.153	.253	.776	.529
10. $\sqrt{U^2+W^2}$	7.42	7.35	7.39	7.58	7.5
11. $(1-S^2)E$	15.5	12.12	13.8	22.55	18.4
12. F_3	.367	.367	.367	.367	.367
13. $M = (11) - (12)$	15.133	11.753	13.433	22.283	18.033
14. $F_9\ U$	.975	.966	.972	.993	.9825
15. $N = (13)+(14)$	16.11	12.719	14.405	23.276	19.0155
16. $\sqrt{N^2+F_4^2}$	23.7	21.5	22.55	29.0	25.7
17. $I_1 = (16)/(10)$	3.19	2.92	3.05	3.83	3.43
18. $(1-S^2)\ F_7$	14.85	11.6	13.2	21.6	17.6
19. $\sqrt{(18)^2+F_5^2}$	17.08	14.25	15.6	23.1	19.5
20. $I_y = (19)/(10)$	2.3	1.94	2.11	3.05	2.6
21. SF_5	8.05	8.12	8.06	7.92	8.0
22. $I_x = (21)/(10)$	1.085	1.105	1.095	1.045	1.067
23. $(1-S^2)F_8$	21700	16900	19260	31400	25700
24. F_6	456	456	456	456	456
25. $(23) - (24)$	21244	16444	18804	30944	25244

Contd....

Table No. 13.3 (Contd...)

26.	Primary Cu. loss $= I_1^2 r_{sm}$	44.4	37.1	40.5	63.7	51
27.	Sec. Cu loss (m) $= I_y^2 r_{rm}$	35	24.8	29.4	61.2	44.5
28.	Sec. Cu loss (c) $= I_x^2 r_{rm}$	7.75	8.05	7.9	7.2	7.5
29.	Core loss (m)	30.24	30.24	30.24	30.24	30.24
30.	$\frac{(25) \times (2)}{(10)^2}$	360	288	324	486	414
31.	Input = (26) + (27) + (28) + (29) + (30)	477.4	388,2	432	648	547
32.	Core loss C + F + W	42.84	42.84	42.84	42.84	42.84
33.	Output = (30) - (32)	317.16	225.14	281.16	443.16	371.16
34.	rpm. = S x Sm rpm	1450	1460	1455	1425	1440
35.	Torque = $112.6 \times \frac{(33)}{(34)}$ oz.ft.	24.6	17.35	21.75	35.0	29.8
36.	Eff. = (33)/(31)	66.5	58.0	65.4	68.4	68.0
37.	P.F. = (31)/EI	43.3	33.5	40.3	49.7	47.2
38.	Apparent eff = (36)/(37)	65.1	57.8	61.6	73.6	69.4
39.	Percent full load	85	60	75	120	100

TABLE NO. 13.4.

Predetermination by Veinott's Method.

Output in watts.	In % of F.L.	Input	Current Amp.	Slip in %	Torque oz.ft.	Effi-ciency	P.F.
0	0	100	2.2	0	0	0	12.0
225.14	60	388.19	2.92	2.7	17.35	58	58
281.20	75	432.0	3.04	3.0	21.75	65.4	65.4
317.16	85	477.4	3.19	3.4	24.6	66.5	66.5
371.2	100	547.24	3.43	4.4	29.8	68.0	69.8
443.2	120	648.3	3.83	5.0	35	67.4	75.0

CHAPTER NO. 14

TESTS & TEST RESULTS.

I. No Load Saturation Test.

Voltage is increased slowly and the no load current and total losses in watts are read for various voltages. The result of this test is given in Table No. 14.1 and the saturation curve is shown in Fig. No. 14.1

II. Measurement of resistance:

The Table 14.2 give the particulars for this, which gives an average resistance of 5.48 ohms. However, immediately after short circuit test when the resistance was measured, it was found to be,

$$r_{sm} = \frac{20.8}{3.5} = 5.95 \text{ ohms.}$$

The resistance of starting winding

$$r_{ss} = \frac{31.2}{2} = 15.6 \text{ ohm.}$$

III. Locked rotor Tests:

This is done with main winding alone across supply at first and with auxiliary winding alone across supply. When the full volt is applied, the spring balance readings are also noted downs. These are given in Table No. 14.3.

IV. Load Test:

The details of the load test carried out are given

in Table No. 14.4 and the performance curves are given in Fig. No. 14.3

V. Heat run Test:

The readings are given in Table No. 14.5. The temperature rise = 37.5°C above 30° ambient.

Temperature by resistance measurement:

The resistance of primary winding after heat run = 6.13 ohms.

Resistance at 35° = 5.45 ohms.

$$\text{Therefore} \quad \frac{r_{35}}{r_{60}} = \frac{1+\alpha 35}{1+\alpha t}$$

$$\therefore \quad \frac{5.45}{6.13} = \frac{1 + .00425 \times 35}{1 + .00425\, t}$$

Solving for t, we get t = 69°

$\therefore$ Temperature rise above ambient temperature of 30°

= 39°.

VI. Insulation Test:

Since D.C. high voltage was not available insulation resistance was measured with a 400 volts meggar and is found to be 20 megaohms.

VII Dielectric Test.

As required by B.S.S. 170 (1939) (with the ammendments) (Please refer chapter No. 11) the insulation of the winding to frame was able to withstand 1000 volts (rms).

VIII. Determination of Constants.

In this the method suggested by Veinott in his paper (J36) "Segragation of losses in a single phase induction motor" will be followed.

For any machine, after conducting certain test it must be necessary to find various losses and certain constants of the machine to verify that whether they confirm with the designed values.

Seggregation of losses in a single phase f- h.p. motor differs from that of polyphase motors in three important respects (1) No load sec. Cu. loss cannot be neglected (2) rotor Cu. loss under load conditions cannot be computed directly from the slip (3) It is necessary to determine certain fundamental constants of the motor. These fundamental constants are x_o, x, r_r where r_r is the rotor resistance. x_o is defined as reactance of primary with secondary open circuited i.e. $x_o = x_m + (x_s + x_r)/2$ and x ideal short circuit reactance;

$$x = x_s + \frac{x_r x_m}{2(x_m + x_r)}$$

These three constants namely x_o, x_s, r_r can be determined from a reading of watts. and amperes at full voltage with rotor locked.

The interpretation of no load and locked readings is slightly more involved than that for polyphase motor. Using cross field theory, the locked rotor current can be obtained as

$$I_L = \frac{V}{(r_s + P) + jQ} \qquad (1)$$

where

$$P = \frac{r_r k_r}{1 + \left(\frac{r_s}{x_o}\right)^2} - \frac{r_r (x_o - x)}{x_o \left[1 + \left(\frac{r_r}{x_o}\right)^2\right]} \qquad (2)$$

$$Q = \frac{x\left(1 + \frac{x_r}{x} \cdot \frac{r_r}{x_o}\right)}{1 + \left(\frac{r_r}{x_o}\right)^2} \qquad (3)$$

and

$$k_r = \sqrt{\frac{x_o - x}{x_o}} \qquad (4)$$

The no load it is given by the equation

$$I_o = \frac{2}{2 - k_2^2} I_m$$

So

$$I_o = \frac{2V}{x_o + x} \qquad (5)$$

In terms of test result, P and Q are given as follows
r_{sm} being measured
directly after the locked readings. $P = \frac{W_L}{I_L^2} - r_{sm}$

$$Q = \sqrt{\frac{V^2}{I_L^2} - \left(\frac{W_L}{I_L}\right)^2} \qquad (6)$$

The most accurate method for determining the desired quantities r_r, x, x_o term the values I_o, P, Q is by simultaneous~~ly~~ solution of equations 2,3, & 5 which yields.

$$x = \frac{V}{I_o} - \sqrt{\left(\frac{V}{I_o} - Q\right)^2 - P^2} \qquad (7)$$

$$x_o = \frac{2V}{I_o} - x \qquad (8)$$

$$t_t = \frac{Px_o}{x_o - a} \qquad (9)$$

Main winding resistance $r_{sm} = \frac{20.8}{3.5} = 5.95$ ohms.

$$P = \frac{W_L}{I_L^2} - r_{sm}$$

$$= \frac{1640}{10.4^2} - 5.95$$

$$= 9.15$$

$$Q = \sqrt{\frac{V_L^2}{I_L^2} - \frac{W_L}{I_L^2}}$$

$$= \sqrt{4950 - 15.2}$$

$$= 21.8$$

Ideal short circuit reactance $x = \frac{V}{I_o} - \sqrt{\left(\frac{V}{I_o} - Q\right)^2 - P^2}$

$$= \frac{230}{2.325} - \sqrt{\left(\frac{230}{2.325} - 21.8\right)^2 - 9.15^2}$$

$$= 22.4$$

$$x_o = \frac{2V}{I_o} - x = 198 - 22.4 = 175.6$$

Secondary resistance referred to main winding

$$r_r = \frac{PX_o}{x_o - Q} = \frac{9.15 \times 175.6}{175.6 - 21.8} = 10.45$$

$$k = \sqrt{\frac{x_o - x}{x_o}} = \sqrt{\frac{175.6 - 22.4}{175.6}} = .9$$

short circuit reactance with respect to main winding.

$$Z_m = \frac{230}{10.4} = 22.2$$

$r_s + r_r$ in terms of main winding $= 16.4$

$x_s + x_r$ in terms of main winding $= \sqrt{22.2^2 - 16.4^2}$

$= 15.2$

$x_s = x_{1m} = 12; \quad x_r = 3.2$

Magnetising reactance $= x_m = x_o - \dfrac{x_s + x_r}{2}$

$$= 175.6 - \frac{15.2}{2}$$

$$= 165.$$

Short circuit reactance with respect to short winding

$$Z_s = \frac{230}{6.5}$$

$$= 35.4$$

$$x_c = 31.8$$

IX. Separation of iron and friction losses.

Both primary and secondary in losses may be subtracted from the running light input to obtain core loss and friction. The secondary copper loss when running light is by using cross field theory

$$I_y^2 r_r + I_x^2 r_r = 2I_x^2 r_r \quad \text{since } I_x = I_y$$

at synchronous speed. Since I_x is not directly measurable it is convenient to express it in terms of primary current I_o. as follows:

$$I_o = \frac{2}{2 - k_r^2} I_m$$

$$I_x = \frac{k_r}{2 - k_r^2}$$

$$\text{So} \quad \frac{I_x}{I_o} = \frac{k_r}{2} \quad \therefore I_x = I_o \frac{k_r}{2}$$

$\therefore$ No. load sec. Cu loss $= .5\, k_r r_r I_o^2$

which is simple convenient equation for practical use. So core loss and friction loss can be separated from the no load input, by subtracting the total copper loss. Friction loss may be separated from core loss by drawing a graph between opp. volt and total core and friction and windage loss and extrapolating the curve to cut at y axis which will give the friction, and windage loss.

$\therefore$ No load sec. copper loss $= .5 \times .9 \times 10.45 \times I_o^2$

Total no load copper loss $= (.45 \times 10.45 + 5.95) I_o^2$

$$= 10.65\ I_o^2$$

For each applied voltage copper loss is calculated and iron and friction loss is calculated by subtracting the copper loss from the total no load loss. This is shown in Table No. 14.6. A graph with voltage in x axis and loss in y axis is drawn and is extrapolated as described earlier. (Fig. No. 14.2) From the graph,

Friction and windage loss = 6 watts.

Iron loss at rated voltage 230 V = 24.5 watts.

X. Starting Torque:

Spring balance readings are 34 lbs. and 16.85 lbs.

$$\text{Starting torque} = 17.5 \times 16 \times 2.98 \times \frac{16}{12}$$

$$= 70 \text{ oz ft.}$$

Full load torque = 33.2 oz.ft.

$$\therefore \text{ ratio of starting to full load torque} = \frac{70}{33.2} = 2.1$$

XI. Break down torque:

At the time of pulling out.

$$\text{Pull out torque} = 13 \times 16 \times 2.98 \times \frac{16}{12}$$

$$= 50 \text{ oz.ft.}$$

Speed = 1250

Input = 900 watts. Current = 5.5

Break down output = .945 h.p.

Ratio of break down to full load torque = $\frac{50}{33.2}$ = 1.5

XII. Discussion:

According to NEMA standards given in Chapter No. 11, the h.p. rating is defined by the value of breakdown torque of the motor and a ½ h.p. motor should give a break down torque between 48.5 oz.ft and 69.5 oz.ft. Our machine gives 50 oz.ft. Hence its h.p. rating may be given as ½ h.p.

The performance curves obtained from tests are also shown along with the curves predetermined with the help of the design constants (Fig. Nos. 14.3 & 14.4) for comparison. The deviations of the forecast performance from the actual performance are mostly due to the empirical formulae that are used in determining the constants of the machine. Further these may be due to imperfections in the actual construction. For example the actual rotor resistance is nearly double the value calculated from design. This may be due to blow holes in the end ring and also due to the imperfect soldering between rotor bars and rings.

References: B14, & J 36.

TABLE NO. 14.1

No Load Saturation Test.

Applied voltage.	No load current I_o	Total loss in watts.
270	3.12	142
260	2.92	130
250	2.74	110.5
240	2.55	99.5
230	2.35	90
220	2.22	80
210	2.09	60
200	1.95	49
190	1.8	42
180	1.66	47
170	1.56	40
160	1.45	36
150	1.34	30
140	1.24	25
130	1.14	20
120	1.05	19
110	.95	15

TABLE NO. 14.2.

Measurement of Resistance.

Applied Voltage.	Current.	Resistance.
16.5	3	5.5
19.2	3.5	5.5
21.75	4.0	5.45

Average Resistance = 5.48

TABLE NO. 14.3

Locked Rotor Test.

	Applied voltage.	Current	Total loss W_L	Sp. Balance Reading T_1	T_2
1. Main winding alone.	230	10.4	1640	-	-
2. Start winding alone	230	6.5	1320	-	-
3. Both windings	230	13.5		34 lb	16.5 lb

LOAD TEST

TABLE NO. 14.4

Diameter of Brake pulley = 2.98".

No.	Voltage	Current	Input in watts	Power factor.	Spring balance readings T_1	T_2	T_1-T_2	Speed in rpm.	Slip in %	Torque oz.ft	Output in B.H.P.	Output in %	Efficiency.
1	230	2.34	90	.152	0	0	0	1496	.266	0	0	0	0
2	230	2.6	290	.328	3	.2	2.8	1470	2.0	11.1	.196	39.2	50
3	230	2.88	360	.4	5.75	1.40	4.35	1450	3.33	13.25	.3	60.0	61
4	230	3.5	505	.629	10	3.40	6.6	1414	5.4	26.2	.445	89	66
5	230	3.05	395	.562	10.25	5.00	5.25	1420	5.33	20.9	.354	70.8	67
6	230	3.3	466	.615	11.0	4.60	6.4	1400	6.66	26.5	.425	85.0	68
7	230	3.25	455	.61	12.25	6.25	6.0	1405	6.33	25.2	.4	80	65.5
8	230	3.32	472	.62	14.25	7.7	6.55	1405	6.65	26.5	.435	87.0	68.8
9	230	3.7	552	.65	16.75	9.0	7.75	1400	6.66	33.2	.490	100	68.2
10	230	3.2	550	.75	12.75	4.9	7.85	1380	8.0	31.8	.515	103	70.0
11	230	3.8	570	.655	13.0	4.9	7.1	1375	8.35	33.2	.465	93.0	61.0
12	230	4.0	630	.682	14.3	6.0	8.3	1355	9.7	38.6	.535	107.0	63.5
13	230	3.9	605	.675	14.0	5.6	8.4	1365	9.0	35.8	.545	109	67.2
14	230	4.0	620	.675	13.75	5.4	8.35	1358	9.45	37.6	.54	108	65.0
15	230	4.2	665	.690	15.6	6.5	9.1	1330	11.3	45.0	.575	115	64.5
16	230	5.47	910	.725	20	8.5	11.5	1340	11.5	47.5	.7	140	58

TABLE NO. 14.5

Heat Run Test

Time hrs	Voltage	Load current	Sp.Balance T_1	T_2	Input watts.	Speed	Temp.	Winding Temp.	Temp. rise.
10.00	230	3.5	12.5	5.5	490	1410	28	36	8
10.30	230	3.5	12.5	5.4	510	1400	28	49.5	21.5
11.00	230	3.5	12.5	5.5	520	1395	29	57.0	28.0
11.30	230	3.5	11.0	4.2	515	1395	29	59.0	30.0
12.00	230	3.75	12.5	4.2	575	1400	29	61.5	32.5
12.30	230	3.4	13.0	5.5	500	1410	29	63.5	34.5
13.00	230	3.6	13.0	5.5	545	1390	30	66.0	36.0
13.30	230	3.65	13.0	5.0	555	1395	30	67.0	37.0
14.00	230	3.2	13.0	5.2	485	1410	30	67.2	37.2
14.30	230	4.0	12.0	5.2	600	1390	30	67.2	37.2
15.00	230	3.7	11.0	4.0	550	1390	30	67.5	37.5
15.30	230	3.6	12.0	3.0	520	1400	30	67.5	37.5
16.00	230	3.5	11.0	3.5	520	1400	30	67.5	37.5

TABLE NO. 14.6

Separation of Core & frictional losses.

Amp. volt.	No load	Total loss.	Copper loss.	friction & winddage loss.
270	3.12	142	103	37
260	2.92	130	90.5	39.5
250	2.74	110.5	80.5	30.5
240	2.55	99.5	69.0	30.5
230	2.35	90	58.5	31.5
220	2.22	80	52.0	28
210	2.09	60	46.5	13.5
200	1.95	49	40.5	8.5
190	1.8	42	24.5	7.7
180	1.6	47	27.2	19.8
170	1.56	40	26.1	13.9
160	1.45	36	22.4	13.6
150	1.34	30	19.2	10.8
140	1.24	25	16.4	8.6
130	1.14	20	13.85	6.15
120	1.05	19	13.4	5.6
110	.95	15	9.6	5.4